A brief syllabus of 21-344 Numerical Linear Algebra

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1 People and contacts

I am Zecheng Zhang. My office is Wean Hall 7216. My email is zechengz@andrew.cmu.edu. The grader of the class is Jiannan Jiang. His office is Wean Hall 6213. My office hours will be announced soon.

2 Books and sources

We do not require a textbook, but the following books are good references.

1. Numerical Linear Algebra, 1st Edition, Lloyd N. Trefethen and David Bau III

2. Linear Algebra and Its Applications, 6th Edition, Lay/Lay/McDonald

3. Matrix Analysis, 2nd Edition by Roger A. Horn, Charles R. Johnson

4. Introduction to Linear Algebra, 5th Edition, Gilbert Strang

5. Matrix Computations (Johns Hopkins Studies in Mathematical Sciences), 3rd Edition, Gene H. Golub, Charles F. Van Loan

We use Canvas, particularly Gradescope in Canvas (for homework). In addition, I create a website for the class. You can also access the class homepage through my personal website in CMU math people.

3 Prerequisite

- Prerequisites: 15-112 and (21-240 or 21-241 or 21-242) and (21-259 or 21-268 or 21-269).

- You need to have some basic knowledge of linear algebra. For example, what is a matrix? What are the elementary row operations? What is the REF of a matrix? How to solve a linear system by REF? How to compute the determinant of a square matrix? How to calculate the inverse of the matrix? What is linear transformation? How to compute the eigenvalues? What is matrix diagonalization? I will provide you with notes to review some fundamental topics, but we may not have time to cover these topics in class.

- Coding in Python, MatLab or C++.

4 Catalog Course Description

Catalog Course Description: An introduction to algorithms pertaining to matrices and large linear systems of equations. Direct methods for large sparse problems, including graph data structures, maximum matchings, row and column orderings, and pivoting strategies. Iterative methods, including Conjugate Gradient and GMRES, with a discussion of preconditioning strategies. Additional topics include the computation of eigenvalues and eigenvectors, condition numbers, the QR and singular value decompositions, and least-squares systems.

An essential aspect of this course is the rigorous mathematics proof. We will discuss proofs of the theorems—for example, low-rank approximation, proper orthogonal decomposition, the convergence of some algorithms, etc. Students should expect to have some 'proof' problems in their homework, exams.

5 Topics

Tentative topics of the class. Some topics may be removed, and some other topics will be added.

1. Fundamentals of linear algebra.

2. Singular value decomposition, low-rank approximation, proper orthogonal decomposition, dynamical mode decomposition.

3. QR factorization, Gram-Schmidt orthogonalization, Householder triangularization, lease squares.

4. Conditioning and condition numbers, stability, stability of Householder triangularization, conditioning of the lease squares.

5. Eigenvalue problems, reduction to Hessenberg or tridiagonal form, Rayleigh quotient, computing SVD.

6. Overview of iterative methods, conjugate gradient method, GMRES, preconditioning.

6 Course grading

6.1 Assessment methods and grade policy

- Four homework (**25%**).

- Three midterm exams (**75%**). The exams are in class.

6.2 Grading criteria

Grading criteria. **I may also curve the class and not follow the Table 1.**

90% above	A
80% − 90%	B
70% − 80%	C
60% − 70%	D
50% − 60%	R (fail)

Table 1: Grading criteria

7 Course design and policy

7.1 Attendance Attendance to all lectures is expected, and students are responsible for materials covered in classes that are missed. The instructor will make every attempt to be helpful to students who miss class meetings due to illness or other unavoidable circumstances.

7.2 Academic integrity Any work you submit in this course must be your own, not copied from a friend, book, online resource, or anywhere else. The student handbook states that Carnegie Mellon's academic integrity policies will be strictly enforced. See here. In particular,

1. Students can discuss homework with their classmates, but everyone should write their own homework independently and write the solutions by themselves.

2. No collaboration on exams.

7.3 Electrical devices

1. Laptops and iPad are allowed in class as long as they don't interfere with the class and affect other people.

7.4 Regrade requests If you think the homework/exam grading is incorrect, **please submit a re-grading request within one week after you receive your graded homework/exam.**

7.5 Accommodations for students with disabilities If you have a disability and have an accommodations letter from the Disability Resources office, I encourage you to discuss your accommodations and needs with me as early in the semester as possible. I will work with you to ensure that accommodations are provided as appropriate. If you suspect that you may have a disability and would benefit from accommodations but are not yet registered with the Office of Disability Resources, I encourage you to contact them at access@andrew.cmu.edu.

7.6 Student wellness Many helpful resources are available on campus, and an essential part of the college experience is learning how to ask for help. Asking for support sooner rather than later is almost always helpful. If you or anyone you know experiences academic stress, complex life events, or feelings like anxiety or depression, we strongly encourage you to seek support. Counseling and Psychological Services (CaPS) is here to help: call 412-268-2922 and visit their website. Consider reaching out to a friend, faculty, or family member you trust for help getting connected to the support that can help. You should always feel welcome to discuss sensitive issues with the course staff, but please bear in mind that (a) we are not counselors or therapists and will not attempt to provide such services (but we will listen to you and help however we can), and (b) we are required to report instances of suspected sexual misconduct to the Deputy Title IX Coordinator for Students.

7.7 Diversity and inclusion The Center for Student Diversity and Inclusion offers resources such as mentoring, advising, and coaching (and much more), particularly for students in historically underrepresented groups and first-generation students. More information can be found on the following website: Center for Student Diversity and Inclusion https://www.cmu.edu/student-diversity/. Phone: (412) 268-2150.

7.8 Discrimination, harassment and sexual misconduct We share a responsibility to make our community one where everyone feels safe and welcome inside and outside the classroom. Discrimination, harassment, and abuse (sexual or otherwise) have no place in this course, at CMU, or anywhere else. Some resources for students regarding discrimination and harassment are as follows.

1. Office of Title IX Initiatives https://www.cmu.edu/title-ix/ (412) 268-7125 / tix@cmu.edu

2. Hazing Prevention & Response https://www.cmu.edu/hazingprevention/

3. CMU University Police https://www.cmu.edu/police/. Phone: (412) 268-2323

4. Anonymous reporting https://www.reportit.net/ (username: tartans, password: plaid). Phone: (877) 700-7050