Exercises on Logical Connectives
Note: This is not to be collected or assigned a grade, just for your benefit. I'd be more than happy to work through any of the problems with you, and to look it over with your graded homework. You may not have the appropriate tools to solve all of the exercises.

Many of the connectives that we have seen are very standard. An interesting question to ask is how many (and which) connectives can we say are a minimal set; that is, given some set of logical connectives (n-ary operations on propositional variables) any statement (truth table) can be expressed.
Fact: As you'd expect, any every statement of propositional calculus can be expressed by the set connectives we learned.

Exercise. The set of connectives we have learned is not minimal.

So what is a minimal set of connectives? First let's learn some more that are pretty useful.

Definition. Given propositional variables A and B we define $A \uparrow B$, read "A NAND B" or in normal person's English "not both A and B" to be

A	B	$A \uparrow B$
$\top$	$\top$	$\bot$
$\top$	$\bot$	$\top$
$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\top$

Similar we define $A \downarrow B$, read"A NOR B" and "neither A nor B" is only true when both are false.

Remark. NAND is sometimes called the Sheffer stroke, and NOR is sometimes called the Pierce Arrow.

Exercise. The following are minimal sets of connectives *Hint: Prove the first one, using the above fact, and then use this to prove the rest*

- $\{\vee, \neg\}$
- $\{\wedge, \neg\}$
- $\{\rightarrow, \bot\}$
- $\{\rightarrow, \not\rightarrow\}$
- $\{\uparrow\}$
- $\{\downarrow\}$

So, this proves that NAND is essential the only logical connective one needs. This is kind of amazing considering how simple it is. Why is this powerful? It's less powerful than one would imagine, although useful. The usefulness comes really from saving time. Forumulas have a **complexity**, and it's often useful to do proofs by induction on the complexity of forumlas. Using the prove above, instead of having to worry about all the different logical connectives, all we have to worry about is a minimal set.