

Assignment 1

Friday May 25, 2012

This is Assignment 1. The problems are arranged in the natural order which we learn the material. You should do as much as you can everyday, but if you don't recognize something that means we probably have learned that material yet.

Remember to show all work, and argue any claim, unless otherwise stated. Getting a correct answer does not suffice

This assignment is due Friday May 25.

1 Puzzles

It is intended for you to complete this section on Monday May 21.

1.1 Return to Smullyan Island

Recall that on the island of Smullyan Island, all people are either Knights or Knaves. Knights always tell the truth, and Knaves always lie.

After visiting the island, you told many of your friends about the amazing time you had. Word spread quickly, and the island has become overrun with tourists! Tourists, like you, can choose to either lie or tell the truth. You return the island, and are more confused than ever!

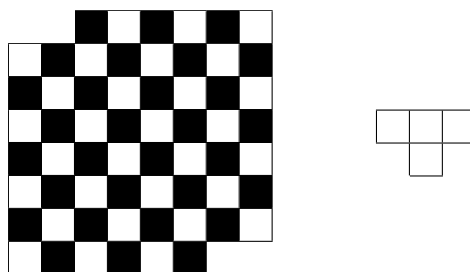
For the following, give your answer and an argument.

Problem 1. You encounter a young child on the island who is repeatedly yelling, "I'm a Knave!" What can you say about the child, aside from the fact he is annoying?

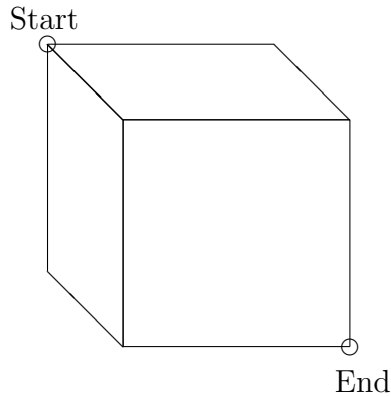
Problem 2. You encounter a group of three people: Albert, Betty, and Catherine. You know one is a knight, another a knave, and another a tourist, although you don't know which is which. Albert tells you that Catherine is a Knave, but Catherine insists she is a tourist. Betty, in order to help clear this confusing situation up, tells you that Albert is a Knight. Who is who?

1.2 More with Chessboards

Problem 3. Take a 8x8 chessboard, and remove two adjacent squares from each corner. Can you cover the board with T-shaped tetrominoes (tetris pieces)? If so, how? If not, why not? (See picture)



Problem 4. A bee is on the corner of a cubic bird house (the front is shown below) at noon. Every 5 minutes, the bee moves from one corner of the bird house to a neighboring corner. He would like to be at the corner of the cube opposite to the one that he started at 12:32. Note that opposite corners of a cube are ones which are across a interior diagonal of the cube. Is it possible? If so, how? If not, why not?



2 Inequalities

It is intended for you to complete this section on Tuesday May 22.

2.1 Understanding

Problem 5. You are planning a party! (No work is necessary for these problems, just the inequalities)

1. Let c be the number of chairs, and t be the number of tables. Write an inequality that says, “We have at least four times as many chairs as tables.”
2. Let p be the number of people at your party. Write an inequality that says, “There are enough tables so that no more than 6 people are at one table.”
3. Write an inequality to relate chairs to tables.

2.2 AGM

Recall that the AGM inequality states that for $x, y \geq 0$ we have that

$$\sqrt{xy} \leq \frac{x+y}{2}$$

Problem 6. Prove the AGM inequality for the arithmetic mean and geometric mean of 4 non-negative reals. That is, prove, for all $a, b, c, d > 0$:

$$\sqrt[4]{abcd} \leq \frac{a+b+c+d}{4}$$

3 Logic

It is intended for you to complete this section on Wednesday May 23.

3.1 Some Proofs

Problem 7. Prove the following:

1. $(A \wedge B) \rightarrow (A \rightarrow B)$
2. $(A \wedge B) \rightarrow (((A \rightarrow C) \vee (B \rightarrow C)) \rightarrow C)$

3. $(A \vee B) \rightarrow (\neg A \rightarrow B)$

Problem 8. Prove the following:

1. $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \wedge B) \rightarrow C)$

2. $((A \wedge B) \rightarrow C) \rightarrow (A \rightarrow (B \rightarrow C))$

3. Consider the following statement:

- If x is a positive real number, then if $y > 1$ we have $xy > x$.

Restate this statement using the logical equivalence proved above. (No work is necessary for this part, just the restatement)

4. Give your own example of a statement which can be rephrased using this equivalence.

4 Logical Semantics

It is intended for you to complete this section on Thursday May 24.

4.1 Contrapositives

Problem 9. Write the contrapositive and converse of the following English sentences. (No work is necessary for these problems, just the sentences)

1. If a person is short then they are not tall.
2. It rains if it pours.
3. Whenever I see a painting by Van Gogh, I think that it is beautiful.
4. I can't take you if the car is full.
5. I can take you if the car isn't full.

4.2 Truth Tables

Problem 10. Write truth tables for Problem 7 to verify they are true (No work is necessary for this problem, just the truth tables)