1. Let $X$ and $Y$ be independent random variables taking values in the set \{0, 1, \ldots\} with the generating functions $G_X$ and $G_Y$. Let $k$ be an integer. Show that $P(X - Y = k)$ equals the coefficient at $t^k$ in the expansion of the function $G_X(t)G_Y(1/t)$ into a formal power series.

2. Let $X_1, X_2, \ldots, X_6$ be independent identically distributed random variables uniform on the set \{0, 1, \ldots, 9\}. Find $P(X_1 + X_2 + X_3 = X_4 + X_5 + X_6)$.

3. Let $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$ be independent random signs. Show that $\text{Var}(\sum_{i=1}^n a_i \varepsilon_i) = \sum_{i=1}^n a_i^2$ for any reals $a_1, \ldots, a_n$.

4. Let $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$ be independent random signs. Show that for any reals $a_1, \ldots, a_n$ we have

$$E\left|\sum_{i=1}^n a_i \varepsilon_i\right|^4 \leq 3 \left(E\left|\sum_{i=1}^n a_i \varepsilon_i\right|^2\right)^2.$$ 

Show that the constant 3 is best possible (in other words, is sharp), that is, if it is replaced with any smaller number, the statement is no longer true.

5. Show that

$$F(t) = \begin{cases} \frac{1}{2}e^t, & t < 0, \\ \frac{1}{2} + \frac{1}{2}(1 - e^{-t}), & t \geq 0 \end{cases}$$

is the distribution function of a random variable, say $X$. Compute $P(X < -1)$, $P(X < 0)$, $P(X \leq 0)$, $P(X = 0)$, $P(X > 1)$ and $P(X = 2)$.

6. The double exponential distribution with parameter $\lambda > 0$ has density $f(x) = \frac{\lambda}{2}e^{-\lambda|x|}$. Find its distribution function, sketch its plot, find the mean, variance and $p$th moment.

7. Let $X$ be a uniform random variable on \(0, 1\). Find the distribution function and density of $Y = -\ln X$. What is the distribution of $Y$ called?

8. Let $X$ be a Poisson random variable with parameter $\lambda$. Show that $P(X \geq k) = P(Y \leq k)$, for $k = 1, 2, \ldots$, where $Y$ is a random variable with the Gamma distribution with parameter $k$.

9. Let $X$ be a random variable with continuous distribution function $F$. Show that $Y = F(X)$ is a random variable uniformly distributed on the interval \(0, 1\).
10. Let $F$ be a distribution function and $U$ be a uniform random variable on $(0, 1)$. Define the generalised inverse of $F$ by

$$G(y) = \inf\{x, \ F(x) \geq y\}.$$  

Show that the distribution function of the random variable $G(U)$ is $F$. 