Question 1. Prove that for any real number \( x \) the following inequality holds
\[ |x + 1| + |x + 2| + \ldots + |x + 2012| \geq 1006^2. \]
When does the equality hold?

Question 2. Find all differentiable functions \( f: \mathbb{R} \rightarrow \mathbb{R} \) satisfying for all \( x \in \mathbb{R} \) the following inequality
\[ f'(x) \geq f(x). \]

Question 3. Let \( \varphi \) be Euler’s totien function, i.e. for a positive integer \( n \) we define \( \varphi(n) \) to be the number of positive integers less than or equal to \( n \) that are relatively prime to \( n \). Prove that for any positive integer \( n \)
\[ \sum_{d|n} \varphi(d) = n, \]
where the sum is over all positive divisors of \( n \).

Question 4. Examine the convergence of the sequence
\[ a_n = \sum_{k=1}^{n} \frac{k}{n^2 + k} = \frac{1}{n^2 + 1} + \frac{2}{n^2 + 2} + \ldots + \frac{n}{n^2 + n}. \]

Question 5. Given an irrational number \( \alpha \) prove that the set \( \{ \{k\alpha\} : k \in \mathbb{Z}\} \) is a dense subset of the interval \([0, 1]\), i.e. prove that for any numbers \( 0 < a < b < 1 \) there exists an integer \( k \) such that \( \{k\alpha\} \in (a, b) \).

Remark. The fractional part, denoted by \( \{x\} \) for real \( x \), is defined by the formula
\[ \{x\} = x - \lfloor x \rfloor, \]
where \( \lfloor \cdot \rfloor \) denotes the usual floor function.

Question 6. Given a parameter \( \beta \in (0, 1) \) prove that
\[ \prod_{k=2}^{n} \left(1 - \frac{\beta}{k}\right) \xrightarrow{n \to \infty} 0. \]

Question 7. Let \( u: (0, 1) \rightarrow \mathbb{R} \) be a differentiable function which satisfies
\[ u'(t) \leq cu(t), \quad \text{for all } t \in (0, 1), \]
where \( c \) is some constant. Prove that
\[ u(t) \leq u(0)e^{ct}, \quad \text{for all } t \in (0, 1). \]

Question 8. Find all integers \( a, b \) and \( c \) satisfying the equation
\[ a^{2012} + b^{2012} - 8c^{1006} = 6. \]

Question 9. Does there exist a non-abelian group with less than 6 elements?

Question 10. Does the series \( \sum_{n=1}^{\infty} (\sqrt[n]{n} - 1)^n \) converge?