1. Suppose 6n fair dice are rolled. Define a probability space that describes this experiment. Let \( p_n \) be the probability that every number (i.e. the numbers 1 through 6) appears exactly \( n \) times in one roll of the collection of dice. Give an exact expression for \( p_n \). Find a simpler function \( f_n \) such that \( p_n \sim f_n \).

2. A woman walks randomly on the \( n \times n \) grid \( \{(x, y) : x, y \in \{1, \ldots, n\}\} \) starting at the point \((1,1)\) (i.e. the lower left corner). Each minute the woman moves either to the right or up (i.e. a move of the form \((a, b) \rightarrow (a + 1, b)\) or a move of the form \((a, b) \rightarrow (a, b + 1)\)). Her walk ends when she reaches the upper right corner, the point \((n, n)\). At each stage in which the woman has a choice of 2 moves she flips a fair coin to determine her next move. (If the woman is on the right edge (i.e. \((x, y)\) such that \(x = n\)) she automatically moves up and if she is on the top edge (i.e. \((x, y)\) such that \(y = n\)) she automatically moves right.) Define a probability space that describes this random walk. What is the probability that the woman reaches the top row of the grid before reaching \((n, n)\)? Explain your answer.

3. Prove that if there is a number \( p \) such that \( 0 \leq p \leq 1 \) and

\[
\binom{n}{k}p^k(1-p)^{\binom{k}{2}} + \binom{n}{t}(1-p)^\binom{t}{2} < 1,
\]

then the Ramsey number \( R(k, t) \) satisfies \( R(k, t) > n \).

4. Suppose we color the edge set of \( K_n \) with 2 colors uniformly at random (so, we have \(|\Omega| = 2^{\binom{n}{2}}\) and the uniform distribution).

(a) Let \( X \) be the number of monochromatic triangles in the random coloring. Determine \( E(X) \).

(b) Conclude that there exists a coloring of the edge set of \( K_n \) that has at most \( \frac{1}{4} \binom{n}{3} \) monochromatic triangles.

5. A collection of \( k \) people enter an elevator in a building that has \( n + 1 \) floors. Each of the \( k \) people has a destination floor that is chosen uniformly at random from floors 2, \ldots, \( n + 1 \). What is the expected number of stops that the elevator makes?

6. Consider the probability space on the finite set \( \Omega \) with the uniform distribution. Let \( X \) be a random variable that takes values in the set \( \{0, 1, 2, \ldots, M\} \) such that \( E[X] = M - a \). Prove that for any \( 1 \leq b \leq M \) we have

\[
P(X \geq M - b) \geq \frac{b - a}{b}.
\]