1. Let the sequence \(a_0, a_1, \ldots\) be defined by \(a_0 = 2, a_1 = 8\) and \(a_i = \sqrt{a_{i-1}a_{i-2}}\) for \(i \geq 2\). Determine \(\lim_{n \to \infty} a_n\).

*Hint: This is a generating functions question.*

2. Prove that any edge coloring of the edge set of \(K_{17}\) with the colors Red, Blue and Green has a monochromatic triangle.

3. Let \(k \geq 3\) and \(n = (k - 1)^2\). Give an explicit 2-coloring of the edges of \(K_n\) that does not have a monochromatic \(K_k\).

4. Prove \(R(3, 5) > 11\).

*Hint: Modify the argument that we used to show \(R(3, 4) > 8\).*

5. We say that a pair of events \(A, B\) in a probability space are independent if

\[
P(A \cap B) = P(A)P(B).
\]

(a) Let \(A\) and \(B\) be independent events in a probability space defined on the set \(\Omega\). Prove that \(\overline{A} = \Omega \setminus A\) and \(\overline{B} = \Omega \setminus B\) are independent events.

(b) Define a probability space with three events \(A, B, C\) with the following properties:

i. \(A\) and \(B\) are independent events,

ii. \(A\) and \(C\) are independent events,

iii. \(B\) and \(C\) are independent events, but

iv. \(P(A \cap B \cap C) \neq P(A)P(B)P(C)\).

6. A women walks randomly on the \(n \times n\) grid \(\{(x, y) : x, y \in \{1, \ldots, n\}\}\) starting at the point \((1, 1)\) (i.e. the lower left corner). Each minute the women moves either to the right or up (i.e. a move of the form \((a, b) \to (a + 1, b)\) or a move of the form \((a, b) \to (a, b + 1)\)). Her walk ends when she reaches the upper right corner, the point \((n, n)\). At each stage in which the woman has a choice of 2 moves she flips a fair coin to determine her next move. (If the woman is on the right edge (i.e. \((x, y)\) such that \(x = n\)) she automatically moves up and if she is on the top edge (i.e. \((x, y)\) such that \(y = n\)) she automatically moves right.) Define a probability space that describes this random walk. What is the probability that the woman reaches the top row of the grid before reaching \((n, n)\)? Explain your answer.