1. Let the sequence \( a_0, a_1, \ldots \) be defined by \( a_0 = 2, a_1 = 8 \) and \( a_i = \sqrt{a_{i-1}a_{i-2}} \) for \( i \geq 2 \). Determine \( \lim_{n \to \infty} a_n \).

   *Hint: This is a generating functions question.*

2. Prove that any edge coloring of the edge set of \( K_{17} \) with the colors Red, Blue and Green has a monochromatic triangle.

3. Let \( k \geq 3 \) and \( n = (k - 1)^2 \). Give an explicit 2-coloring of the edges of \( K_n \) that does not have a monochromatic \( K_k \).

4. If \( G = (V, E) \) is a graph and \( v \in V \) then the *degree* of \( v \), denoted \( d(v) \), is the number of edges in \( G \) that contain \( v \) (e.g. the degree of every vertex in the complete graph \( K_n \) is \( n - 1 \)).
   
   Let \( n \geq 2 \) be an integer. Does there exist a graph with vertex set \( V = \{v_1, \ldots, v_n\} \) such that \( d(v_i) = i - 1 \) for \( i = 1, \ldots, n \)?

5. A graph \( G = (V, E) \) is **bipartite** if there exists a partition \( V = A \cup B \) such that

   \[ E \cap \binom{A}{2} = \emptyset \quad \text{and} \quad E \cap \binom{B}{2} = \emptyset. \]

   In other words, every edge has one vertex in \( A \) and one vertex in \( B \). The sets \( A \) and \( B \) are the parts of the bipartition of \( G \).

   A graph \( G = (V, E) \) is **\( d \)-regular** if every vertex in \( G \) has degree \( d \).

   Let \( G \) be a \( d \)-regular bipartite graph with parts \( A, B \). Prove that if \( d \geq 1 \) then \( |A| = |B| \).

6. We say that a pair of events \( A, B \) in a probability space are **independent** if

   \[ \mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B). \]

   (a) Let \( A \) and \( B \) be independent events in a probability space defined on the set \( \Omega \).
   
   Prove that \( \overline{A} = \Omega \setminus A \) and \( \overline{B} = \Omega \setminus B \) are independent events.

   (b) Define a probability space with three events \( A, B, C \) with the following properties:

   i. \( A \) and \( B \) are independent events,

   ii. \( A \) and \( C \) are independent events,

   iii. \( B \) and \( C \) are independent events, but

   iv. \( \mathbb{P}(A \cap B \cap C) \neq \mathbb{P}(A)\mathbb{P}(B)\mathbb{P}(C) \).