1. Find all positive integers \( a > b > c \) for which
\[
\binom{a}{b} \binom{b}{c} = 2 \binom{a}{c}.
\]
Give a combinatorial justification for your answer (i.e. appeal to the fact that \( \binom{n}{k} \) gives the number of subsets of an \( a \)-element set having exactly \( b \) elements).

2. If \( m \) indistinguishable 4-sided die are rolled how many distinguishable outcomes are there? (E.g. if \( m = 2 \) then there are 10 possibilities.) How many outcomes are there in which each of the four numbers appear at least once?

3. A function \( f : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\} \) is monotone if \( i < j \) implies \( f(i) \leq f(j) \). How many monotone functions \( f : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\} \) are there?

4. Let \( T \) be a set such that \( |T| = n \). How many sequences \( S_1 \subseteq S_2 \subseteq \cdots \subseteq S_k \) such that \( S_i \subseteq T \) for \( i = 1, \ldots, k \) are there?

5. Let \( F \) be a collection of \( k \)-element subsets of the set \( X \), where \( |X| = n \). The shadow of \( F \) is defined to be the set
\[
\partial F = \left\{ B \in \binom{X}{k-1} : \exists A \in F \text{ such that } B \subset A \right\}.
\]
Show
\[
|\partial(F)| \geq \frac{k|F|}{n - k + 1}.
\]
Hint. Count ordered pairs \( (A, B) \) such that \( |A| = k, |B| = k - 1, B \subset A \) and \( A \in F \).

6. Use a combinatorial argument to prove the following: for all positive integers \( m_1, m_2, n \) we have
\[
\sum_{k=0}^{n} \binom{m_1}{k} \binom{m_2}{n-k} = \binom{m_1 + m_2}{n}.
\]