On Stability for an Inverse Hyperbolic Problem

KiHyun Yun
Michigan State University
kyun@math.msu.edu

Abstract: In this talk, we focus on the inverse problem of determining the potential q by the Neumann to Dirichlet map Λ_q in the wave equation $u_{tt} - \Delta u + qu = 0$ in $\Omega \times (0, T)$ with $u(x, 0) = u_t(x, 0) = 0$. Here, we establish a Hölder type stability with exponent $1 - \epsilon$ as follows: for any small $\epsilon > 0$, there exists $\beta_0 > 0$ such that

$$\|q_1 - q_{2L^\infty(\Omega)} \leq C\|\Lambda_{q_1} - \Lambda_{q_2*}^{\;1-\epsilon}$$

when $\|q_1 - q_{2H^\beta(\mathbb{R}^n)} \leq M$ for some $\beta > \beta_0$. Here, $\|\cdot_*$ represents the operator norm.

To mention the previous related work, twenty years ago (1988), using Sylvester and Uhlmann s method that is well known in inverse problems, Ziqi Sun, and Alessandrini and Sylvester obtained $1/3 - \epsilon$ Holder stability

$$\|q_1 - q_{2L^\infty(\Omega)} \leq C\|\Lambda_{q_1} - \Lambda_{q_2*}^{\;1/3-\epsilon}.$$

There has been no remarkable improvement in this result, even though Yamamoto made some results under the strong condition.

As it mentioned in the abstract, we have obtained a nearly Lipschitz type stability estimate under the same condition as Sun.