DGM

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Abstract: In the first part of this presentation we investigate higher-order discontinuous Galerkin methods (DGM) for hyperbolic problems on triangular meshes. A detailed description of the formulation of the DGM on a hyperbolic partial differential equation is presented. We study the effect of finite element spaces on the superconvergence properties of DG solutions on three types of triangular elements. We show that the DG solution is $O(h^{p+2})$ superconvergent at Legendre points on the outflow edge on triangles having one outflow edge using three polynomial spaces. For triangles having two outflow edges the finite element error is $O(h^{p+2})$ superconvergent at the end points of the inflow edge for an augmented space $\mathcal{U}_p$. Furthermore, we discovered additional supeconveregence points in the interior of triangles. We also established a global superconvergence result on meshes consisting of triangles of type III only. Several numerical simulations are performed to validate the theory. Superconvergence on more general meshes as well as *a posteriori* error estimates based on these superconvergence results are presented in the second part of this presentation. We apply these superconvergence results to construct simple, efficient and asymptotically correct *a posteriori* error estimates for discontinuous finite element solutions of scalar first-order hyperbolic partial differential problems on triangular meshes in smooth solution regions. Based to these superconvergence results, we explicitly write the basis functions for the leading term of the error corresponding to several finite element spaces and types of elements and construct asymptotically correct *a posteriori* error estimates by solving local hyperbolic problems with no boundary conditions on more general meshes. The leading term of the discretization error on each triangle is estimated by solving a local problems where no boundary conditions are needed. The computed error estimates are shown to converge to the true error under mesh refinement in smooth solution regions. We also show global superconvergence for discontinuous solutions on general unstructured meshes. The error estimates are tested on several problems, both linear and nonlinear, to show their efficiency, accuracy and asymptotic convergence of the error estimates under mesh refinement.