Asymptotic Convex Optimization for Packing Random Malleable Demands in Smart Grid

Gennady Shaikhet*, Mohammad M. Karbasioun†, Evangelos Kranakis‡, Ioannis Lambadaris†
Carleton University, Ottawa, ON, Canada
*School of Mathematics and Statistics {gennady}@math.carleton.ca
†Department of Systems and Computer Engineering {mkarbasi, ioannis}@sce.carleton.ca
‡School of Computer Science {kranakis}@scs.carleton.ca

***Abstract*—We consider a problem of supplying electricity to a network of electric demands, $\mathcal{N}$, in a smart-grid framework. Each demand requires a random amount of electrical energy which has to be supplied during the time interval $[0, 1]$. We model this network by *malleable rectangular shape* demands, and then relate the resulted scheduling problem to well known Strip Packing problem. in this model, we assume that each demand i has to be supplied without interruption, with possible duration between ℓ_i and r_i, which are given malleability constraints of that demand ($\ell_i \leq r_i$). At each moment of time, the power of the grid is the sum of all the consumption rates for the demands being supplied at that moment. Our goal is to find a scheduling policy that minimizes the *power peak* - maximal power over $[0, 1]$ - and/or total operational convex cost of the system while satisfying all the demands. To find such a policy, we first present an asymptotic analysis of stochastic demands to find the proper tight lower bounds for both types of costs in the system. Eventually we will propose an on-line scheduling algorithm for demands with stochastic energy demands and stochastic malleability constraints and show that the presented algorithm is asymptotically optimum and has fully linear running time.**

I. Introduction

A. Motivations

Wisely designing and implementing a scheduling policy plays a crucial role in resource allocation problems. In the smart grid, the common resource is the available electrical energy at any time. Resource allocation is the main job of what is called Demand Side Management (DSM) [1]. The main goal of DSM is enhancing the efficiency of the grid network while reducing the total cost of using the (limited) resources of that network. This is usually done by smart exploiting some statistical characteristics of (stochastic) demands and then shaping the load profile of the system as much as the natural or logistical constraints permit. In optimization language there are two main cost functions to be optimized in electrical networks. The first one is the total convex cost of using the resources and the second one is the Peak to Average Ratio (PAR) of energy consumption rate of the system. Even tough these two costs are somehow related to each other, they are not the same. The cost of consuming the energy is considered to be a convex function of instantaneous total power consumption due to the fact that each additional unit of power needed to serve demands is more expensive as the total power demand increases [2]. This is because the supplementary power for serving the demands when the consumption rate is high, is generally produced from expensive natural non-sustainable resources e.g. fossil fuels.

In general DSM has several benefits for the grid such as reducing the amount of additional supplementary power needed to satisfy the demand during peak hours which itself results in decreasing the CO_2 emissions of power plants [3]. Also DSM may result in reducing the possibility of power outage due to sudden increase of demands. Furthermore with the expected presence of plug-in hybrid electric vehicles (PHEVs) to the market, during charging hours of PHEVs, the average household load is expected to be doubled [4]. Specially for efficiently serving the upcoming PHEVs to the the market, it is vital to design and deploy suitable charging stations with proper scheduling scheme [5]. Consequently a scheduling policy plays a crucial role in enhancing the utilization of an electrical network.

There are many works in the field of Demand Side Management. As examples for online sxheduling we can refer to [2] and [6], in which by using some queueing analysis techniques, it is attempted to devise on-line scheduling schemes for the random arriving demands with flexibility of delaying the start of their service. Specially in [2], by using a threshold policy, they proposed an asymptotic scheduling policy to minimize the total convex cost of the consumed power in the system. In their setting the running time of the system is assumed to be infinite as well as the acceptable delay for starting the serving of each demand. In [7] it is assumed that demands have different stochastic power requirements and durations and then authors try to propose an on-line energy storage control policy that minimizes long-term average convex operational cost of the grid, in the presence of a storage unit. On the other hand in [8] authors propose a distributed algorithm using game theory for serving the deterministic demands. In this model the power network consists of demands each with its own energy demand and with its own minimum and maximum acceptable power level that should be scheduled in their own requested time intervals. The primary goal of their algorithm is reducing the total convex cost of the network, while they still show that their algorithm can be used to reduce the PAR in the network. The problem in [8] is *off-line* Scheduling of deterministic demands, since complete knowledge of the demands, such as number of demands and the amount of energy needed by each of them are known in advance. Other scenarios can happen when the complete knowledge of the demands is not available and also we may need to perform *on-line* scheduling. For example in [9] authors try to find algorithms for different levels of knowledge about the demands such as arrival times, durations and power intensities. Again the goal is reducing the total convex cost of the power consumption in the system.

In this paper we consider an asymptotic setting with a large number of stochastic small malleable energy demands, which will be defined precisely in subsection I-B. This model itself is novel in the smart grid literature. We will relate this model to Strip Packing problem [10] with some modifications. Then we introduce two types of costs, i.e. the total convex operational cost and the Peak to Average Ratio (PAR) of consuming power of the system and find proper lower bound for these costs. Finally we will propose a proper scheduling policy and then by performing asymptotic stochastic analysis, we will show the proposed policy is asymptotically optimum for both types of costs.

B. Model

Consider a set $\mathcal{N} = \{1, 2, ..., n\}$ of energy demands $\{A_i, i \in \mathcal{N}\}$, needed to be scheduled in a finite time interval $[0, 1]$. We assume that only "rectangular" shape of scheduling is permitted, meaning that each demand $i \in \mathcal{N}$ has to be supplied without interruption in some interval $[\tau_i, \tau_i + s_i]$ with a constant power intensity $d_i = \frac{A_i}{s_i}$. Obviously,

$$0 \le \tau_i \le \tau_i + s_i \le 1, \qquad i \in \mathcal{N}. \tag{1}$$

In addition to (1), we impose a *demand malleability* constraint. That is, we assume each demand has its own parameters ℓ_i and r_i with $0 \le \ell_i \le r_i \le 1$ so that

$$\ell_i \le s_i \le r_i, \qquad i \in \mathcal{N}. \tag{2}$$

This is motivated by the existence of electric appliances with flexibility on the charging rate. We are interested in evaluating the asymptotic performance of the system, i.e. when the number of demands, $n = |\mathcal{N}|$, is large and on the other hand the values of demands, (A_i), are relatively small. As an example we may think of a PHEV charging facility with a significantly large amount of relatively much smaller orders.

The triples (A_i, ℓ_i, r_i), $i \in \mathcal{N}$ will become random i.i.d. vectors, with a certain distribution of (ℓ, r). Again in PHEV charging facility example, it is reasonable to assume that different customers have different energy demands and malleability constraints but with some known distribution on (A_i, ℓ_i, r_i), $i \in \mathcal{N}$.

A set of pairs $\pi = \{(\tau_i, s_i), i \in \mathcal{N}\}$, satisfying (1)–(2) for the given set of demands with the parameters (A_i, ℓ_i, r_i), where $i \in \mathcal{N}$ and $(\ell, r) \in \Omega$, will be called a *scheduling policy*. Let Π be a set of all policies.

For a policy $\pi \in \Pi$, with $(\tau_i, s_i) \in \pi$, we have

$$P^\pi(t) = \sum_{i=1}^{n} \left(\frac{A_i}{s_i} \cdot 1_{\{\tau_i \le t \le \tau_i + s_i\}} \right), \;\; 0 \le t \le 1. \tag{3}$$

Let us introduce two types of costs:

$$C_1^\pi = P_{max}^\pi = \left(\max_{t \in [0,1]} \{P^\pi(t)\} \right), \tag{4}$$

and, for arbitrary convex function h

$$C_2^\pi = \left(\int_{t=0}^{1} h\Big(P^\pi(t)\Big) dt \right). \tag{5}$$

The corresponding optimum costs of (4) and (5) are computed as follow:

$$P_{opt} = \inf_{\pi \in \Pi} P_{max}^\pi \tag{6}$$

$$C_{opt} = \inf_{\pi \in \Pi} C_2^\pi. \tag{7}$$

The problem of finding these optimum values is known to be NP complete (see [11]) and therefore an optimal height or convex cost cannot always be computed in polynomial time. Therefore instead of finding these optimum values and policies resulting in these values, we are interested in finding a scheduling policy (π^*) that is asymptotically optimum with respect to either height (using equations (4) and (6))

$$\text{a.s.} \;\; \lim_{n \to \infty} \left[\max_{t \in [0,1]} \{P^{\pi^*}(t) \right] = P_{opt}, \tag{8}$$

or convex cost (using equations (5) and (7))

$$\text{a.s.} \;\; \lim_{n \to \infty} \left[\int_{t=0}^{1} h\Big(P^{\pi^*}(t)\Big) dt \right] = C_{opt}. \tag{9}$$

C. Related literature

Our setting resembles a so-called strip-packing problem [12]. Specially in the case of the height problem, by viewing the demands as rectangles, we want to pack them with their side s_i parallel to horizontal axis in a rectangular bin of width $[0, 1] \times P_{opt}$ where an optimal height P_{opt} is unknown. As it was mentioned before, the problem is known to be NP complete and therefore an optimal height cannot always be computed in polynomial time.

Strip packing problem (also called two-dimensional strip packing) is a variant of the bin-packing problem ([13] and [14]) in which rectangles are packed into a strip of width 1 and infinite height in a way that rectangles don't overlap each other. Strip packing has been extensively explored in the literature (see [11] and [15] for reviewing some of the important works in this area). The objective in many works related to Strip Packing is minimizing the height of the packing in the strip, i.e. the height problem. The value of the demands can be considered as random values with a certain distribution or as deterministic values. Then the performance of a scheduling policy π in both cases can be measured by worse *case analyses* [16]. Also for stochastic demands, we can use *average case analyses* [17].

Also scheduling policies are mainly divided into two categories, on-line and off-line algorithms. An algorithm is called on-line, when demands arrive one by one and then a demand A_i is scheduled without knowledge of next demands, i.e. $A_{i+1}, \cdots A_n$ [13]. For off-line scheduling problem, most commonly referred work is [18] which has an asymptotic worst case performance ratio $(1 + \epsilon)$ while its running time is polynomial in both n (the number of demands) and $(1/\epsilon)$.

On the other hand [19] proposes an on-line scheduling policy with the same performance ratio for on-line problem while the running time is just linear in the number of demands, n and worse than exponential in $(1/\epsilon)$.

The most related works in strip packing literature to our work is [20] and [19]. In [20], authors generalize the setting of [18] to find an off-line scheduling policy for Malleable Tasks where each task could use different number of resources (e.g. processor, memories,...) which can also alternate their service times. On the other hand in [19], authors propose a on-line scheduling policy for malleable demands where demands can be lengthened while their areas remain fixed. In their setting the height of each rectangular demand is below bounded by its initial height but does not have upper bound for the heights of the rectangles. The values of demands (their width and heights) are unknown but deterministic. The main drawback of this algorithm is the running time in term of $(1/\epsilon)$.

The scheduling problem in this paper (which is called Power Strip Packing (PSP) in the sequel) is different from strip packing problem (we call it Traditional Strip Packing (TSP)) in some aspects. In TSP, the height H_t^π at any time t, for a scheduling policy π, is defined as the *uppermost boundary* of scheduled rectangles at time t, while in PSP the height of the strip packing at time t is obtained from equation (3). This difference arises from the nature of the electric power, in which the overall height (i.e. power) at any given time is just the sum of the scheduled (i.e. active) demands (See Figure 1). Naturally, $P_t^\pi \leq H_t^\pi$ for any policy π and consequently $P_{max}^\pi \leq H_{max}^\pi$. Another difference which also arises from the nature of power demands, is that demands can overlap each other and then the amount of consumed power at each time is the sum of all scheduled demands at that time.

D. Our Contributions

In this paper, we address the problem of optimal power demand scheduling subject to malleability constraints to minimize either the total convex cost or the Peak to Average Ratio of the power grid. The contribution of our work to the literature of smart grid and Strip Packing is as follows:

- We model a network of electric demands by *malleable rectangular shape* demands, and then relate the resulted scheduling problem to well known Strip Packing problem.
- In addition to minimizing the maximum height of the scheduling, our problem aims at minimizing the total convex cost of the consumption power (height) in the network.
- We present an asymptotic analysis of stochastic demands to find the proper tight lower bounds for two types of costs in the system, i.e. maximum height and total convex cost.
- Eventually we will propose an on-line scheduling algorithm for demands with stochastic energy demands and stochastic malleability constraints and show that the presented algorithm is asymptotically optimum and also has fully linear running time.

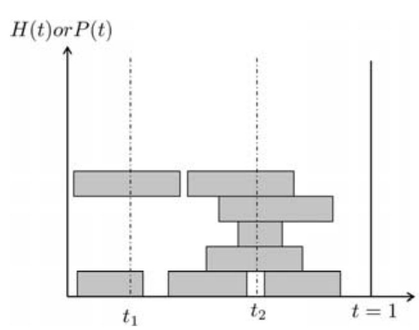

Fig. 1: Different interpretations of power and height in PSP and TSP. For simplicity the height of each rectangle is assumed to be 1. In PSP: $P(t_1) = 2$ and $P(t_2) = 4$ where in TSP $H(t_1) = H(t_2) = 5$.

The paper is organized as follows: In the next section after introducing the asymptotic setting of the system, the lower bounds of two different criteria are calculated. Then a scheduling policy will be proposed which will be further shown to be asymptotically optimal. Finally in the section III some simulation results will be illustrated.

II. Asymptotic Optimization

A. Random Setting

In this section we consider an asymptotic setting with a large number of stochastic small malleable energy demands, scheduling a significantly large amount of relatively much smaller orders. Consider a set $\mathcal{N} = \{1, 2, ..., n\}$ of "rectangular" shape energy demands $\{A_i, i \in \mathcal{N}\}$, needed to be scheduled in a finite time interval $[0, 1]$. In addition, we assume each demand has its own parameters ℓ_i and r_i with $0 \leq \ell_i \leq r_i \leq 1$ so that $\ell_i \leq s_i \leq r_i$. The triples (A_i, ℓ_i, r_i), $i \in \mathcal{N}$ will become random i.i.d. vectors, with a certain distribution of (ℓ, r). Assume that the pair of demand constraints (ℓ, r) is distributed uniformly in a region

$$\Omega = \{(\ell, r) \; : \; 0 \leq \ell \leq r \leq 1\},$$

This would imply that a likelihood for a demand to have r as a right constraint would be directly proportional to r, since

$$(L, R) \sim \text{Unif}\,(\Omega) \;\Rightarrow\; \text{Prob}[R \in (r, r + dr)] = 2r dr. \quad (10)$$

By the Law of Large Numbers this would make the number of demands with a certain right constraint r to be *directly* proportional to its value. That would be a *natural* situation, where the system changes extra for short ("express") right constraints, with all left constraints being equal to a constant, *as small, as possible*, just to satisfy technical requirements of a charging facility.

B. Model and Results

For convenience, as well as to address possible industry applications of the method, we will replace Ω by its discrete version. For this we take an arbitrary natural number $d > 1$ (which would be set by a charging system), and define

$$\Omega_d = \left\{ \left(\frac{i}{d}, \frac{j}{d} \right) \; : \; i, j \in \{1, ..., d-1\}, \; i \leq j \right\} \quad (11)$$

Assume that we are given random vectors (ℓ_i, r_i), $i \in \mathcal{N}$, i.i.d., distributed uniformly in Ω_d. Set

$$p_k := \text{Prob}\left[r_1 = \frac{k}{d}\right]. \tag{12}$$

Easy algebra gives us

$$p_k = \frac{2k}{d(d-1)}. \tag{13}$$

Thus implying a beautiful fact, that we will use extensively, that the relation p_k over k remains constant:

$$\frac{p_k}{k} = \frac{2}{d(d-1)}, \qquad k = 1, ..., d-1. \tag{14}$$

Now we are ready to introduce an asymptotic regime, emphasizing the large number of demands and small sizes for each demand. Suppose n is large, and there is another large number $m(n)$, directly proportional to n with some coefficient $\lambda > 0$:

$$\frac{n}{m(n)} \to \lambda \ , \quad \text{as } \ n \to \infty. \tag{15}$$

Let $\{\xi_i\}$, $i = 1, ..., n$ be a sequence of i.i.d. random variables with $\alpha := E(\xi_1) < \infty$ and $\sigma^2 := Var(\xi_1) < \infty$. For the distribution of the sizes of the demands we assume

$$A_i = (\text{in distribution}) = \frac{\xi_i}{m(n)}, \tag{16}$$

which reflects the fact that demand sizes are small.

Recall the definition (3) for the function $P^\pi(t)$, as well as the definition for a scheduling policy π, with respect to malleability constraint (2). Note that Π - a set of all possible policies, depends on n, as well as on (A_i, ℓ_i, r_i), $i = 1, ..., n$. Now recall two types of costs defined in (4) and (5) which we rewrite them here:

$$C_1^\pi = P_{max}^\pi = \left(\max_{t\in[0,1]}\{P^\pi(t)\}\right), \tag{17}$$

and, for arbitrary convex function h

$$C_2^\pi = \left(\int_{t=0}^1 h\Big(P^\pi(t)\Big)dt\right). \tag{18}$$

and their corresponding optimum values:

$$P_{opt} = \inf_{\pi\in\Pi} P_{max}^\pi. \tag{19}$$

$$C_{opt} = \inf_{\pi\in\Pi} C_2^\pi. \tag{20}$$

For the set of demands, $\mathcal{N}$, with $\mid \mathcal{N} \mid= n$, the following asymptotic lower bound L_P^n for P_{opt} easily follows from (15), (16):

$$P_{opt} \geq \sum_{i=1}^n A_i = L_P^n. \tag{21}$$

Also to find the corresponding lower bound L_C^n for C_{opt}, one needs to apply Jensen's inequality to get

$$C_{opt} \geq h\left(\int_{t=0}^1 P^\pi(t)\right)dt = h\left(\sum_{i=1}^n A_i\right) = L_C^n. \tag{22}$$

Now in asymptotic regime when $n \to \infty$, by using the Law of Large Numbers, we have:

$$\sum_{i=1}^n A_i = \sum_{i=1}^n \frac{\xi_i}{m(n)} \to \lambda\alpha \ \text{ a.s. } \ n \to \infty. \tag{23}$$

Then using (23), the asymptotic values for L_P^n and L_C^n, become as follows:

$$L_P^n \to \lambda\alpha \ \text{ a.s. } \ n \to \infty. \tag{24}$$

$$L_C^n \to h(\lambda\alpha) \ \text{ a.s. } \ n \to \infty. \tag{25}$$

To state the main result of the section, we first need to introduce the scheduling policy π^*, which we call it *Grouping Policy*.

Algorithm 1 Grouping Policy π^*

INPUT (A_i, ℓ_i, r_i) where $(\ell_i, r_i) \in \Omega$ and $i = 1, \ldots, n$
OUTPUT Scheduling parameters (τ_i, s_i) where $i = 1, \ldots, n$
if $r_i < \frac{1}{2}$ **then**
  set $\tau_i = 0, s_i = r_i$
  (i.e., schedule the demand in the interval $[0, r_i]$);
else if $r_i > \frac{1}{2}$ **then**
  set $\tau_i = 1 - r_i, s_i = r_i$,
  (schedule the demand in the interval $[1 - r_i, 1]$);
else if $r_i = \frac{1}{2}$ **then**
  alternatively set either $\tau_i = 0, s_i = 1/2$, or $\tau_i = 1/2, s_i = 1/2$,
  (schedule in $\left[0, \frac{1}{2}\right]$ or $\left[\frac{1}{2}, 1\right]$ alternatively).
end if

Note the *online* nature of π^* - the system does not need to know anything about the other demands, each demand can be scheduled, once its information is accessed.

Theorem 1: A policy π^* is asymptotically optimal, in the sense that the lower bounds (21)–(22) are asymptotically achieved.

$$\text{a.s.} \quad \lim_{n\to\infty}\left[\max_{t\in[0,1]}\{P^{\pi^*}(t)\right] = \lim_{n\to\infty} L_P^n \tag{26}$$

$$\text{a.s.} \quad \lim_{n\to\infty}\left[\int_{t=0}^1 h\Big(P^{\pi^*}(t)\Big)dt\right] = \lim_{n\to\infty} L_C^n \tag{27}$$

Proof: In what follows we omit the superscript π^* to simplify the notation. Decompose the set of all demands into $\lfloor \frac{d}{2} \rfloor$ groups, in the following fashion:

$$Q_k = \begin{cases} \left\{i \in \mathcal{N} \ : \ r_i = \frac{k}{d} \text{ or } 1 - \frac{k}{d}\right\}, & k = 1, ..., \lfloor \frac{d-1}{2} \rfloor; \\ \left\{i \in \mathcal{N} \ : \ r_i = \frac{1}{2}\right\} & k = \frac{d}{2}. \end{cases} \tag{28}$$

Next, introduce the functions P_k, for $k = 1, .., \lfloor \frac{d}{2} \rfloor$ as

$$P_k(t) = \sum_{i\in Q_k}\left(\frac{A_i}{s_i} \cdot 1_{\{\tau_i \leq t \leq \tau_i + s_i\}}\right), \ \ 0 \leq t \leq 1. \tag{29}$$

Comparing with (3), we look at P_k as a total electricity demand over a group k. Obviously,

$$P^{\pi^*}(t) = \sum_k P_k(t). \tag{30}$$

From the definition of the scheduling policy π^*, each P_k will be a piecewise constant, with only two possible values

$$P_k(t) = \begin{cases} P_{k,1} & 0 \le t \le \frac{k}{d}, \\ P_{k,2}, & \frac{k}{d} < t \le 1, \end{cases} \tag{31}$$

where

$$P_{k,1} = \sum_{i \in Q_k} \left(\frac{A_i}{s_i} \cdot 1_{\{\tau_i \le t \le \tau_i + s_i\}} \right), \; 0 \le t \le \frac{k}{d}, \tag{32}$$
$$P_{k,2} = \sum_{i \in Q_k} \left(\frac{A_i}{s_i} \cdot 1_{\{\tau_i \le t \le \tau_i + s_i\}} \right), \; \frac{k}{d} < t \le 1.$$

The main step would be to prove the following a.s. limits

$$\lim_{n\to\infty} P_{k,1} = \lim_{n\to\infty} P_{k,2} = \lambda\alpha \, \frac{2}{d-1}, \qquad \text{for } \; k = 1, ..., \lfloor \frac{d-1}{2} \rfloor, \tag{33}$$

$$\lim_{n\to\infty} P_{k,1} = \lim_{n\to\infty} P_{k,2} = \lambda\alpha \, \frac{1}{d-1}, \qquad \text{for } \; k = \frac{d}{2}. \tag{34}$$

Given the above, the theorem, namely (26)–(27), will follow from (30) and a simple algebraic relation

$$\left\lfloor \frac{d-1}{2} \right\rfloor \lambda\alpha \, \frac{2}{d-1} + \lambda\alpha \, \frac{1}{d-1} \cdot 1_{\{\text{d is even}\}} = \lambda\alpha, \tag{35}$$

as well as from the smoothness of a convex function h.

We start by proving (33). For $k = 1, ..., \frac{d}{2} - 1$, decompose Q_k into a unit of disjoint $Q_{k,1}$ and $Q_{k,2}$:

$$Q_{k,1} = \left\{ i \in \mathcal{N} \;\; : \;\; r_i = \frac{k}{d} \right\}; \qquad k = 1, ..., \lfloor \frac{d-1}{2} \rfloor \tag{36}$$
$$Q_{k,2} = \left\{ i \in \mathcal{N} \;\; : \;\; r_i = 1 - \frac{k}{d} \right\}.$$

Using the above together with (32), we have (see the explanation below)

$$P_{k,1} = \sum_{i \in Q_{k,1}} \frac{A_i}{r_i} = (\text{distribution}) = \sum_{i \in Q_{k,1}} \frac{d}{k} \, \frac{\xi_i}{m} \tag{37}$$
$$= \left(\frac{|Q_{k,1}|}{n} \right) \left(\frac{d}{k} \, \frac{n}{m} \right) \left(\frac{1}{|Q_{k,1}|} \sum_{i \in Q_{k,1}} \xi_i \right)$$
$$\to p_k \, \frac{d}{k} \, \lambda \, \alpha \;\; = \;\; \lambda\alpha \, \frac{2}{d-1}.$$

Here we used Law of Large Numbers for for the convergence of the first and third brackets; the relation (15) for the second bracket, as well as the relation (14). Similar convergence holds for $P_{k,2}$:

$$P_{k,2} = \sum_{i \in Q_{k,2}} \frac{A_i}{r_i} = (\text{distribution}) = \sum_{i \in Q_{k,2}} \frac{d}{d-k} \, \frac{\xi_i}{m} \tag{38}$$
$$\to p_{d-k} \, \frac{d}{d-k} \, \lambda \, \alpha \;\; = \;\; \lambda\alpha \, \frac{2}{d-1},$$

thus proving (33). The only difference intreating the case $k = \frac{d}{2}$, is that, instead (36), we need to set

$$Q_{\frac{d}{2},1} = \{ i \in \mathcal{N} \;\; : \;\; \tau_i = 0 \} ; \tag{39}$$
$$Q_{\frac{d}{2},2} = \left\{ i \in \mathcal{N} \;\; : \;\; \tau_i = \frac{1}{2} \right\}.$$

Using the fact that, by the policy π^*, we must have (with a negligible difference)

$$|Q_{\frac{d}{2},1}| = |Q_{\frac{d}{2},2}| = \frac{1}{2} |Q_{\frac{d}{2}}|, \tag{40}$$

we repeat the lines (38) as to get (34), and hence (35). Therefore we have:

$$\text{a.s.} \;\; \lim_{n\to\infty} \left[\max_{t \in [0,1]} \{ P^{\pi^*}(t) \right] = \lambda\alpha, \tag{41}$$

$$\text{a.s.} \;\; \lim_{n\to\infty} \left[\int_{t=0}^{1} h\Big(P^{\pi^*}(t) \Big) dt \right] = h(\lambda\alpha). \tag{42}$$

Finally combining the equations (24)-(25) with (41)-(42), with respect to definitions (4) and (5)), results in the following equations:

$$\text{a.s.} \;\; \lim_{n\to\infty} |P^{\pi^*}_{max} - L^n_P| \le \lim_{n\to\infty} |P^{\pi^*}_{max} - \lambda\alpha| + |L^n_P - \lambda\alpha| = 0 \tag{43}$$
$$\text{a.s.} \;\; \lim_{n\to\infty} |C^{\pi^*}_2 - L^n_C| \le \lim_{n\to\infty} |C^{\pi^*}_2 - h(\lambda\alpha)| + |L^n_C - (\lambda\alpha)| = 0 \tag{44}$$

This concludes the theorem. ■

III. Computational Results

Figures 2 and 3 illustrate the outputs of the Grouping Policy π^*, for different values of n when $d = 9$ and $\lambda = 1$ and ξ_i's are i.i.d Gaussian random variables with $\alpha = 10$ and $\sigma^2 = 4$. The cost function h is assumed to be: $h(x) = x^4$. As it is illustrated in these figures with increasing the number of demands the performance of π^* gets better which is consistent with the asymptotic nature of thus policy. In figure 2, to illustrate the performance of the Grouping Policy π^* with respect to Peak to Average ratio criteria, i.e. $C^{\pi^*}_1 = P^{\pi^*}_{max}$ (equation (17)), two different ratios are computed: theoretical asymptotic height ratio and real height ratio are calculated. The theoretical asymptotic height ratio is the proportion of the $P^{\pi^*}_{max}$ over theoretical asymptotic lower bound $\lambda\alpha$ (equation (24)) and the real height ratio is the

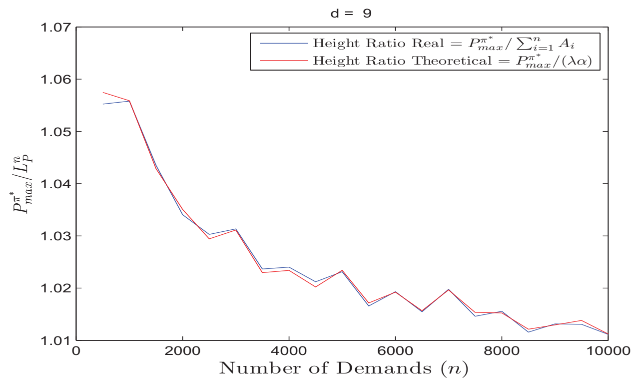

Fig. 2: The performance of the policy π^* with respect to Peak to Average Ratio criteria

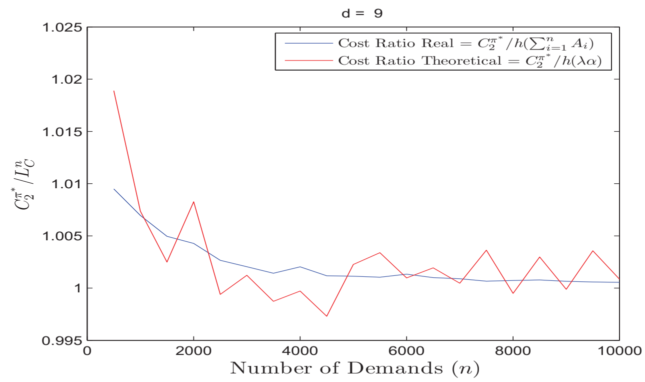

Fig. 3: The performance of the policy π^* with respect to total operational convex cost of the system.

proportion of the $P^{\pi^*}_{max}$ over real lower bound $L^n_P = \sum_{i=1}^n A_i$ (equation (21)). Similarly figure 3 illustrates the performance of the Grouping Policy π^* with respect to total convex cost criteria, i.e. $C^{\pi}_2 = \left(\int_{t=0}^1 h\left(P^{\pi}(t)\right)dt\right)$ (equation (18)), with respect to theoretical asymptotic and real cost ratios. So in figure 3, the theoretical asymptotic cost ratio is the proportion of the $C^{\pi^*}_2$ over theoretical asymptotic lower bound $h(\lambda\alpha)$ (equation (25)) and the real height ratio is the proportion of the $P^{\pi^*}_{max}$ over real lower bound $L^n_C = h(\sum_{i=1}^n A_i)$ (equation (22)).

IV. Conclusion

In this paper considering a problem of supplying electricity to malleable demands, we introduced Power Strip Packing (PSP) problem. With respect to two cost criteria, power peak and total operational convex cost of the system we showed that using a linear time algorithm will result in asymptotically optimal performance. The main contribution of this work is to propose a *deterministic* scheduling policy for supplying stochastic demands with stochastic malleability constraints and show that this policy is asymptotically optimum. It should be noted that, even though we focused on PSP in this paper, our analysis and results are also valid for traditional strip packing problem. One possible extension of this work is proposing a scheduling policy for more general distribution on vectors (ℓ_i, r_i), which we are currently working on it and we will propose the more general *random* scheduling policy in our future works.

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