Practice Midterm 2

July 23, 2008

Note: This practice midterm doesn't have the format of the midterm. It is simply a collection of problems you should know how to solve. Some of them could have been on the midterm, as I was considering them, but for one reason or another, didn't include.

Problem 1 *Does the following define an inner product on M_{22}?*

$$A \cdot B = trace(A)trace(B).$$

(The trace of a square matrix is the sum of its diagonal entries)

Problem 2 *Let $T : M_{22} \to \mathbb{R}_2$ be the linear transformation given by*

$$A \mapsto (trace(A), sum\ of\ the\ elements\ of\ A).$$

Find the matrix that gives this linear transformation if we choose the standard bases for M_{22} and $\mathbb{R}^2$.

Problem 3 *Find the equation of the line that best fits the given points $(-2,-2), (-1,0), (0,-2), (1,0)$ in the least-squares sense.*

Problem 4 *Find the projection of the vector $(1,2,3)$ and the zx plane in $\mathbb{R}^3$. What is the distance from $(1,2,3)$ to the zx plane?*

Problem 5 *Show that $S = \{u_1, u_2, u_3\}$ is an orthonormal basis for $\mathbb{R}^3$, where $u_1 = (2/3, 2/3, 1/3), u_2 = (\sqrt{2}/2, -\sqrt{2}/2, 0), u_3 = (\sqrt{2}/6, \sqrt{2}/6, -2\sqrt{2}/3)$. Using the fact that S is an orthonormal basis, find the coordinates of $v = (1,1,1)$ in this basis.*

Problem 6 *Apply the Gram-Schmidt process to the vectors*

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}.$$

The inner product is $A \cdot B = a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + a_{22}b_{22}$.

Problem 7 *Let $A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$. Find A^{25}.*

Problem 8 *Find an orthonormal basis of* $\mathbb{R}^3$ *consisting of eigenvectors of* $A = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & -2 \\ 2 & -2 & 0 \end{pmatrix}$.

Problem 9 *Find the eigenvalues and eigenvectors of the linear transformation* $T : M_{22} \to M_{22}$ *given by* $A \mapsto A^T$.

Problem 10 *Let* A *be a* 4×4 *matrix,* u_1, u_2 *linearly independent vectors in* $\mathbb{R}^3$. *Suppose* $Au_1 = Au_2 = 0$. *Which of the following are true?*

1. *A can not be invertible.*
2. $dim(CS(A)) = 2$.
3. $rk(A) \leq 2$.
4. *A has at most 3 distinct eigenvalues.*
5. *A is not diagonalizable.*

Problem 11 *The characteristic polynomial of* A *is* $-\lambda^3 - \lambda$. *Which of the following is true?*

1. *A is definitely diagonalizable?*
2. *A is definitely not diagonalizable?*
3. *A may or may not be diagonalizable?*

Problem 12 *(True or False) If* $x \cdot y = 0$ *for all* $y \in \mathbb{R}^3$ *then* x *must be* $\bar{0}$.

Problem 13 *Let* $T : C[-1, 1] \to C[-1, 1]$ *be the linear transformation given by* $(T(f))(x) = f(-x) \forall x \in [-1, 1]$. *Find the eigenvectors and eigenvalues of* T.