Practice Midterm 2

Note: This practice midterm is not as close to the actual midterm as the first practice midterm was to the first midterm. Some of the problems here were on the midterm, but because of size limitations, were taken out.

1. *Is the map* $T : M_{22} \to P_2$ *given by* $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \mapsto (a+(b+c)x+dx^2)$ *a linear transformation?*

2. *Does the following define an inner product on* M_{22}*?*

$$A \cdot B = trace(A)trace(B).$$

3. *Let* $T : M_{22} \to \mathbb{R}^2$ *be the linear transformation given by*

$$A \mapsto (trace(A), \textit{sum of the elements of } A)$$

Find the matrix that gives this linear transformation if we choose the standard bases for M_{22} *and* $\mathbb{R}^2$*.*

4. *(This one is for "entertainment") Show that for any real numbers* $a_1, b_1, c_1, a_2, b_2, c_2$ *the following inequality always holds:*

$$(a_1a_2 + b_1b_2 + c_1c_2)^2 \leq (a_1^2 + b_1^2 + c_1^2)(a_2^2 + b_2^2 + c_2^2).$$

When will the inequality actually be an equality?

5. *Find the equation of the line that best fits the given points* $(-2, -2), (-1, 0), (0, -2), (1, 0)$ *in the least-squares sense.*

6. *Find the projection of the vector* $(1, 2, 3)$ *on the zx plain in* $\mathbb{R}^3$*. What is the distance from* $(1, 2, 3)$ *to the zx plain?*

7. *Show that* $S = \{u_1, u_2, u_3\}$ *is an orthonormal basis for* $\mathbb{R}^3$*, where* $u_1 = (2/3, 2/3, 1/3), u_2 = (\sqrt{2}/2, -\sqrt{2}/2, 0), u_3 = (\sqrt{2}/6, \sqrt{2}/6, -2\sqrt{2}/3)$*. Using the fact that S is an orthonormal basis, find the coordinates of* $v = (1, 1, 1)$ *in this basis.*

8. *Apply the Gram-Schmidt process to the vectors*

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}.$$

The inner product is $A \cdot B = a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + a_{22}b_{22}$*.*
When applying Gram-Schmidt, I suggest doing normalization at the very end. This simplifies the intermediate calculations a lot. To do this you have to use the right version of Gram-Schmidt. Try this on the quiz 8 problem.

9. *Find* $\det\left(\begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 1 & 2 & 3 \\ 3 & 0 & 1 & 2 \\ -1 & 2 & 0 & 4 \end{bmatrix}\right)$

10. *Prove that* $\det(A^{-1}) = \det(A)^{-1}$.

11. (10pts) *Let* $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$. *Find* A^{25}.

12. *Find an orthonormal basis of* $\mathbb{R}^3$ *consisting of eigenvectors of* $A = \begin{bmatrix} 0 & 2 & 2 \\ 2 & 0 & -2 \\ 2 & -2 & 0 \end{bmatrix}$.

13. *Find the eigenvalues and eigenvectors of the linear transformation* $T : M_{22} \to M_{22}$ *given by* $A \mapsto A^T$.

14. *Let* A *be a* 4×4 *matrix,* u_1, u_2 *linearly independent vectors in* $\mathbb{R}^3$. *Suppose* $Au_1 = Au_2 = 0$. *Which of the following are true?*

    (a) A *can not be invertible.*

    (b) $\dim(CS(A)) = 2$.

    (c) $rk(A) \leq 2$.

    (d) A *has at most* 3 *distinct eigenvalues.*

    (e) A *is not diagonalizable.*

15. *The characteristic polynomial of* A *is* $\lambda^3 + \lambda$. *Which of the following is true?*

    (a) A *is definitely diagonalizable.*

    (b) A *is definitely not diagonalizable.*

    (c) A *may or may not be diagonalizable.*

16. *(True or False) If* $x \cdot y = 0$ *for all* $y \in \mathbb{R}^3$ *then* x *must be* $\bar{0}$.