

## Group-work problems. Lec 05.

- 1.
2. *Determine if  $V$  is closed under addition, scalar multiplication, both, or neither.*
  - a.  $V = \{\text{odd integers}\}.$
  - b.  $V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \text{ are integers} \right\}.$
  - c.  $V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \text{ are integers} \right\}.$
  - d.  $V = \{f \in C[1, 2] : f(x) \geq 0 \forall x \in [1, 2]\}.$
3. *Let  $V$  be the set of all solutions  $y(x)$  to the differential equation  $y'' - 4y' = 0$ .*
  - a. Is 0 in  $V$ ?
  - b. Is  $V$  closed under addition, scalar multiplication?
  - c. Is  $V$  a vector space?
4. In a vector space  $V$ , show that if  $u + 0' = u$  for all vectors  $u$  in  $V$ , then  $0' = 0$  (the zero vector). This means that this property of the zero vector uniquely characterizes it.
5. Let  $P_n$  be the set of polynomials of degree at most  $n$ . Show that  $P_n$  is a vector space under the usual addition and multiplication by constants. Is this true for the set of all polynomials of degree exactly  $n$ ?