Math 54 Summer 2006

Group-work problems. Lec 04.

1. Answer the following True or False. If True, explain your reasoning; if False, give a counterexample.

   - If A and B are $n \times n$ matrices such that $AB = 0$, then $A = 0$ or $B = 0$.
   - If A is an $n \times n$ matrix such that $A^2 = 0$, then $A = 0$.

2. *Suppose x is a real number satisfying $x^2 = 1$. To solve for x, we factor $x^2 - 1 = (x-1)(x+1) = 0$, and conclude that $x = \pm 1$. What if X is a 2×2 matrix satisfying $X^2 = I$?*

   - Show that $(X - I)(X + I) = 0$.
   - Let a be any real number, and let $A = \pm \begin{bmatrix} 1 & 0 \\ a & -1 \end{bmatrix}$. Show that $A^2 = I$.
   - So for matrices we found infinitely many solutions to $X^2 = I$. Where does the analogy between x and X break down?

3. a. Compute the inverse of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$.

   b. Use part (a) to solve the system $\begin{cases} x_1 + 2x_2 + 3x_3 = & 5 \\ 2x_1 + 5x_2 + 3x_3 = & 3 \\ x_1 + 8x_3 = & 17 \end{cases}$

4. Determine whether the following matrix is invertible. $\begin{bmatrix} 2 & 1 & -3 & 1 \\ 0 & 5 & 4 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$. (Hint: don't try to calculate the inverse. Think about systems of linear equations).

5. a. Show that the equation $AX = X$ can be written as $(A - I)X = 0$, where A is an $n \times n$ matrix, I the $n \times n$ identity matrix and X is an $n \times 1$ matrix.

   b. Use part (a) to solve $AX = X$ for X, where $\begin{bmatrix} 2 & 2 & 3 \\ 1 & 2 & -1 \\ 2 & -2 & 1 \end{bmatrix}$.

   c. Solve $AX = 4X$.

6. Suppose that $A^2 - 3A + I = 0$ Show that $A^{-1} = 3I - A$.