## Math 21-325 - Probability

## Part of Homework Assignment 8 Due Nov 2

- 1. Consider a random walk where the probability of going up at any step is  $p < \frac{1}{2}$ . What is the probability that the walk will return to the origin infinitely many times?
- 2. Prove that the first Borel-Cantelli lemma holds under the following weaker condition. Prove that if the events  $A_1, A_2, \ldots$  satisfy the conditions

$$\lim_{n \to \infty} P(A_n) = 0$$

and

$$\sum_{n=1}^{\infty} P(A_n \cap A_{n+1}^c) < \infty,$$

then

 $P(A_n \text{ occurs infinitely often}) = 0.$ 

3. (Bonus problem) Prove that the second Borel-Cantelli lemma holds, even if the events are (only) pairwise independent. I.e. Prove, that if the events  $A_1, A_2, \ldots$  are pairwise independent and

$$\sum_{i=n}^{\infty} A_n = \infty,$$

then

 $P(A_n \text{ occurs infinitely often}) = 1.$