Math 21-325 - Probability

Part of Homework Assignment 10
Due Nov 16

1. Read Section X.5.

2. A random variable X is called exponentially distributed with parameter $\alpha > 0$ iff it has a probability density function
$$f_X(t) = \alpha e^{-\alpha t} 1_{[0,\infty)}(t).$$
Check that
$$\int_{-\infty}^{\infty} f_X(t)dt = 1$$
and compute the expected value of X.

3. So far we have only discussed the standard normal distribution. In general, we say X is normally distributed with parameters (μ, σ^2) iff
$$f_X(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma}}.$$
The normal distribution is usually denoted by $N(\mu, \sigma^2)$.

   Prove, that if X is distributed according to $N(0, 1)$, then $Y = \sigma X + \mu$ is distributed according to $N(\mu, \sigma^2)$. Using this, compute $E(Y)$ and $Var(Y)$.

4. Suppose the random variables X and Y have joint density function
$$f_{X,Y} = \frac{1}{2\pi} e^{-\frac{(x-1)^2+y^2}{2}}.$$
Check that
$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x, y)dydx = 1$$
and compute
$$P(|(X, Y) - (1, 0)| \le R).$$