Math 371 - Lie Theory

Homework Assignment 8 Due Nov 9

- 1. Let $Sp(2n, \mathbb{C})$ be the symplectic Lie group corresponding to the symplectic form $(u, v)_s = u^T J v$. Since $Sp(2n, \mathbb{C})$ is a subgroup of $GL(2n, \mathbb{C})$ the Lie algebra $sp(2n, \mathbb{C})$ of $Sp(2n, \mathbb{C})$ consists of matrices from $gl(2n, \mathbb{C})$. Describe the Lie algebra $sp(2n, \mathbb{C})$. (Note: I mentioned in class that perhaps you should determine what the Lie bracket is. There is no need to do that. It will be standard Lie bracket for matrix Lie algebras).
- 2. Find the dimension of $sp(2n, \mathbb{R})$ over \mathbb{R} .
- 3. Find the dimension of $sp(2n, \mathbb{C})$ over \mathbb{R} .
- 4. Determine whether $sp(2n, \mathbb{C})$ is a complex Lie algebra or not. Justify your answer.
- 5. Show that $sp(2n, \mathbb{C})$ is the complexification of $sp(2n, \mathbb{R})$.