1. Important Problems

(1) State Wilson's Theorem and Fermat's Little Theorem. Define what it means for u to be the multiplicative inverse of x mod n. Define what it means for k to be the order of x mod n.
(2) Let $n \in \mathbb{N}$. Show that n is divisible by 5 if and only if the ones digit of n is either 0 or 5.
(3) Solve the congruence $x \equiv 10! \mod 13$.
(4) Solve the congruence $x \equiv 8! \mod 70$
(5) Find the multiplicative inverse of 5 mod 13.
(6) Compute the order of 4 mod 13.
(7) Prove that a 6-digit number of the form *abcabc* (for example, 153153) is always disible by 11.
(8) What is the last digit of 9^{100}?
(9) What is the remainder of 353^{52} divided by 11?
(10) Let $m, n, p \in \mathbb{Z}$. Suppose that 5 divides $m^2 + n^2 + p^2$. Prove that 5 divides at least one of m, n, p. (Hint: What are the possible congruence classes of x^2 mod 5?)
(11) Let x and y be integers with at least k digits. Show that if the last k digits of x and y are the same, then 5^x divides x if and only if 5^k divides y.

For example 125 divides 2875, so we can conclude that 125 also divides 2384875.

2. Extra Problems

(1) Suppose that n is odd. Show that the sum of any n consecutive integers is divisible by n.
(2) Is $2^{100} + 1$ prime?
(3) Prove that $n(n^2 - 1)(n^2 - 2)(n^2 + 3)$ is divisible by 7 for any $n \in \mathbb{N}$.
(4) Prove that in any right triangle with integer lengths, one of the lengths must be a multiple of 3.
(5) Prove that the only integer solution to the equation $n^2 = 5m^2$ is $(0, 0)$