1. Important Problems

1.1. Sets.

(1) Compute the following:
  (a) $\mathcal{P}(\{0, \emptyset, \{1, 2\}\})$
  (b) $\mathcal{P}(\emptyset, \{\emptyset\})$

(2) Prove that if A and B are sets, then $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.
  Is this still true if we replace $\cap$ with $\cup$?

(3) For each $n \in \mathbb{N}$, let $A_n = [-n-1, -1] \cup [1, n+1] = \{x \in \mathbb{R} \mid -n-1 \le x \le 1\} \cup \{x \in \mathbb{R} \mid 1 \le x \le n+1\}$. Compute

$$\bigcap_{n \in \mathbb{N}} (\mathbb{R} \setminus A_n).$$

1.2. Induction.

(1) Show for each natural number n that $\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$.

(2) Show that $3n \le 2^n$ for all natural numbers $n \ge 4$.

(3) ~~Show that for every natural number n, $7^{n+1} + 8^{2n+1}$ is divisible by 57.~~

(4) Show that if $\{a_i\}_{i=0}^{\infty}$ is the sequence satisfying $a_1 = 1$, $a_2 = 13$ and $a_n = a_{n-1} + 6a_{n-2}$ for $n \ge 3$. Show that

$$a_n = (-2)^n + 3^n.$$

2. Extra Problems

(1) For a given set X, what are the following?
  (a) $\bigcup_{A \in \mathcal{P}(X)} A$
  (b) $\bigcap_{A \in \mathcal{P}(X)} A$

(2) Give an example of sets A and B such that $\mathcal{P}(A) \Delta \mathcal{P}(B) \not\subseteq \mathcal{P}(A \Delta B)$.

(3) Show that the sum of angles of a convex n-gon is $(n-2)180$ degrees. A polygon is *convex* if there is no straight line segment between two points on the boundary ever goes outside the polygon.

(4) Prove that for all $n, k > 0$, $F_{n+k} = F_{n+1}F_k + F_nF_{k-1}$ where F_n are the Fibonacci numbers defined by $F_1 = 1$, $F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n > 2$.