Please follow the instructions and guidelines detailed at the beginning of the first assignment.

This homework is out of 100pts.

(1) [10 pts] Let $n$ be a modulus and let $a \in \mathbb{Z}$ such that $a \neq 0, 1, -1$. Show that there exists $k > 0$ such that $a^k \equiv 1 \mod n$ iff $a$ and $n$ are relatively prime. (We showed the $\Leftarrow$ direction in class. Show the $\Rightarrow$ direction).

(2) [20 pts] Let $p$ be an odd positive prime. Prove that

$$\left( \frac{p-1}{2} \right)! \equiv (-1)^{\frac{p+1}{2}} \mod p.$$ 

(3) [10 pts] Find all integers $x$ such that

$$25x - 4 \equiv 4x + 3 \mod 13.$$ 

(4) [10 pts] Prove that the only integer solution to the equation $n^3 = 40m^3$ is $(0, 0)$.

(5) [20 pts] Show that in every set of 7 distinct integers, there is a pair whose sum or difference is a multiple of 10

*Hint: pigeonhole principle*

(6) [20 pts] Let $n \in \mathbb{N}$. If $n$ is of the form $d_r d_{r-1} ... d_1 d_0$ we say that the alternating sum of the digits of $n$ is the sum

$$d_0 + (-d_1) + d_2 + (-d_3) + ... + (-1)^r d_r.$$ 

Show that $n$ is divisible by 11 if and only if the alternating sum of it’s digits are divisible by 11.

(7) [10 pts] You want to get an accurate count of the number of people at a party.

- If you ask everyone to make groups of 5, there are 3 left over.
- If they make groups of 12, there are 4 left over.
- If they make groups of 7, there are 3 left over.

You know there are somewhere between 500 and 1000 people. Exactly how many are there?

*You will not be able to do this problem until Monday, unless you read the textbook*