Please follow the instructions and guidelines detailed at the beginning of the first assignment.

This homework is out of 100pts.

(1) **[10 pts]** Let n be a modulus and let $a \in \mathbb{Z}$ such that $a \neq 0, 1, -1$. Show that there exists $k > 0$ such that $a^k \equiv 1 \mod n$ iff a and n are relatively prime. (We showed the $\Leftarrow$ direction in class. Show the $\Rightarrow$ direction).

(2) **[20 pts]** Let p be an odd positive prime. Prove that

$$\left[\left(\frac{p-1}{2}\right)!\right]^2 \equiv (-1)^{\frac{p+1}{2}} \mod p.$$

(3) **[10 pts]** Find all integers x such that

$$25x - 4 \equiv 4x + 3 \mod 13.$$

(4) **[10 pts]** Prove that the only integer solution to the equation $n^3 = 40m^3$ is $(0, 0)$.

(5) **[20 pts]** Show that in every set of 7 distinct integers, there is a pair whose sum or difference is a multiple of 10

Hint: pigeonhole principle

(6) **[20 pts]** Let $n \in \mathbb{N}$. If n is of the form "$d_r d_{r-1} ... d_1 d_0$" we say that the alternating sum of the digits of n is the sum

$$d_0 + (-d_1) + d_2 + (-d_3) + ... + (-1)^r d_r.$$

Show that n is divisible by 11 if and only if the alternating sum of it's digits are divisible by 11.

(7) **[10 pts]** You want to get an accurate count of the number of people at a party.

- If you ask everyone to make groups of 5, there are 3 left over.
- If they make groups of 12, there are 4 left over.
- If they make groups of 7, there are 3 left over.

You know there are somewhere between 500 and 1000 people. Exactly how many are there?

You will not be able to do this problem until Monday, unless you read the textbook