Please follow the instructions and guidelines detailed at the beginning of the first assignment.

This homework is out of 100pts.

(1) [10 pts] Let $n \in \mathbb{N}$ such that $n \geq 2$. Define relation $R$ on $\mathbb{Z}$ by $xRy$ iff $x$ and $y$ have the same remainder when divided by $n$. Show that $R$ is an equivalence relation. What is the corresponding partition of $\mathbb{N}$?

(2) [10 pts] Define a relation $E$ on $\mathcal{P}(\mathbb{R})$ by $XEY$ iff $X \cap Z = Y \cap Z$. Is $E$ an equivalence relation? Prove your answer. What is the corresponding partition of $\mathcal{P}(\mathbb{R})$? What is $[\{2n \mid n \in \mathbb{Z}\}]_E$?

(3) [10 pts] We say that $x$ and $y$ are relatively prime if their greatest common divisor is 1. Define a relation $S$ on $\mathbb{Z} \setminus \{0\}$ by $xSy$ iff $x$ and $y$ are relatively prime. Is $S$ an equivalence relation? Prove your answer.

(4) [10 pts] We define a relation $F$ on $\mathbb{R}$ by $xFy$ iff $x - y \in \mathbb{Q}$. Is $F$ an equivalence relation? Prove your answer.

(5) [10 pts] Let $a$ and $b$ be integers. Show that if $d$ and $d'$ are greatest common divisors of $a$ and $b$, then either $d = d'$ or $d = -d'$.

This problem explains why we can refer to the greatest common denominator of $a$ and $b$.

(6) [10 pts] Use the Euclidean Algorithm to compute the gcd of -72 and 252. Show your work.

(7) [20 pts] For each of the following linear diophantine equations, determine if it has an integer solution or not. If it has an integer solution, specify all of them. If it does not have an integer solution, prove it.

(a) $630x + 385y = 4340$
(b) $453x + 48y = 20$
(c) $104x + 598y = 0$

(8) [10 pts] Let $a$ and $b$ be integers, both not zero, and let $d = \gcd(a, b)$. Show that $\frac{a}{d}$ and $\frac{b}{d}$ are relatively prime.

(9) [10 pts] Let $a, b \in \mathbb{Z}$. Prove that if $\gcd(ab, a + b) = 1$ then $\gcd(a, b) = 1$. 
