Please follow the instructions and guidelines detailed at the beginning of the first assignment.

This homework is out of 100pts.

(1) **[10 pts]** Len $n \in \mathbb{N}$ such that $n \geq 2$. Define relation R on $\mathbb{Z}$ by xRy iff x and y have the same remainder when divided by n. Show that R is an equivalence relation. What is the corresponding partition of $\mathbb{N}$? What is $[3]_R$?

(2) **[10 pts]** Define a relation E on $\mathcal{P}(\mathbb{R})$ by XEY iff $X \bigcap \mathbb{Z} = Y \bigcap \mathbb{Z}$. Is E an equivalence relation? Prove your answer. What is the corresponding partition of $\mathcal{P}(\mathbb{R})$? What is $[\{2n \mid n \in \mathbb{Z}\}]_E$?

(3) **[10 pts]** We say that x and y are *relatively prime* if their greatest common divisor is 1. Define a relation S on $\mathbb{Z} \setminus \{0\}$ by xSy iff x and y are relatively prime. Is R an equivalence relation? Prove your answer.

(4) **[10 pts]** We define a relation F on $\mathbb{R}$ by xFy iff $x - y \in \mathbb{Q}$. Is F an equivalence relation? Prove your answer.

(5) **[10 pts]** Let a and b be integers. Show that if d and d' are greatest common divisors of a and b, then either $d = d'$ or $d = -d'$.

This problem explains why we can refer to *the* greatest common denominator of a and b.

(6) **[10 pts]** Use the Euclidean Algorithm to compute the gcd of -72 and 252. Show your work.

(7) **[20 pts]** For each of the following linear diophantine equations, determine if it has an integer solution or not. If it has an integer solution, specify *all* of them. If it does not have an integer solution, prove it.

   (a) $630x + 385y = 4340$

   (b) $453x + 48y = 20$

   (c) $104x + 598y = 0$

(8) **[10 pts]** Let a and b be integers, both not zero, and let $d = \gcd(a, b)$. Show that $\frac{a}{d}$ and $\frac{b}{d}$ are relatively prime.

(9) **[10 pts]** Let $a, b \in \mathbb{Z}$. Prove that if $\gcd(ab, a + b) = 1$ then $\gcd(a, b) = 1$.