

Please follow the instructions and guidelines detailed at the beginning of the first assignment.

This homework is out of 50pts.

- (1) [10 pts] Every year, a committee of 4 math professors meet to organize the year's parties. Over the course of 20 years, in which no professor leaves and no new professor arrives, the same committee never meets more than once. How many professors must there be? Prove your answer.
- (2) [20 pts] Along the edge of a circle, the numbers 1 through 13 are arranged. Prove that no matter what order the numbers appear, there must be a set of four consecutive numbers that sum up to at least 30. Prove that this is best possible by exhibiting an ordering such that there is no set of four consecutive numbers that sum up to at least 31.
- (3) [10 pts] Recall that if $n \geq 1$ is a natural number then $[n] = \{1, 2, \dots, n\}$. A permutation of $[n]$ is a bijection $f : [n] \rightarrow [n]$. We say that $x \in [n]$ is a fixed point of f if $f(x) = x$. How many permutations are there of $[7]$ without an even number as a fixed point? Prove your answer.
- (4) [10 pts] How many natural numbers less than 250 do not have a divisor in $\{6, 9, 15\}$? Prove your answer.