

Please follow the instructions and guidelines detailed at the beginning of the first assignment.

This homework is out of 50pts.

(1) **[20 pts]** Let X be a non-empty set. Show that the following are equivalent.

- (a) X is countable
- (b) There exists a countable set C and a surjection $f : C \rightarrow X$.
- (c) There exists a countable set C and an injection $f : X \rightarrow C$.

Note the slight difference from the theorem we covered in lecture. You don't need to re-do that proof!

You can prove this stronger theorem by combining the weaker version with other results.

Would it be correct to add the following statements?

- (d) There exists a countable set C and an injection $f : C \rightarrow X$.
- (e) There exists a countable set C and a surjection $f : X \rightarrow C$.

(2) **[15 pts]** Show that for each $n \in \mathbb{N}$, $\mathcal{P}([n])$ is countable. (Hint: Use induction)

(3) **[15 pts]** Show that $\mathcal{P}(\mathbb{N})$ is uncountable.