Please follow the instructions and guidelines detailed at the beginning of the first assignment.

This homework is out of 50pts.

(1) [20 pts] Let $X$ be a non-empty set. Show that the following are equivalent.
   (a) $X$ is countable
   (b) There exits a countable set $C$ and a surjection $f : C \to X$.
   (c) There exists a countable set $C$ and an injection $f : X \to C$.

   Note the slight difference from the theorem we covered in lecture. You don’t need to re-do that proof!
   You can prove this stronger theorem by combining the weaker version with other results.

   Would it be correct to add the following statements?
   (d) There exists a countable set $C$ and an injection $f : C \to X$.
   (e) There exists a countable set $C$ and a surjection $f : X \to C$.

(2) [15 pts] Show that for each $n \in \mathbb{N}$, $\mathcal{P}([n])$ is countable. (Hint: Use induction)

(3) [15 pts] Show that $\mathcal{P}(\mathbb{N})$ is uncountable.