Please follow the instructions and guidelines detailed at the beginning of the first assignment.

This homework is out of 100pts.

(1) **[20 pts]** Let $f : X \to Y$ and $g : Y \to Z$. For each of the following, prove your answer.
  (a) If f is injective, can we conclude that $g \circ f$ is injective?
  (b) If g is surjective, can we conclude that $g \circ f$ is surjective?
  (c) If f and g are both surjective, can we conclude that $g \circ f$ is surjective?
  (d) If f and g are both injective, can we conclude that $g \circ f$ is injective?

(2) **[20 pts]** Let $f : A \to B$ be a function. Let S and T be subsets of A. If f is injective, can we conclude that $f[S \setminus T] = f[S] \setminus f[T]$? Prove your answer.

(3) **[10 pts]** Let $p : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be the function $p(x, y) = 2^x(2y + 1) - 1$. Show that p is a bijection.

(4) **[10 pts]** Find the inverse of the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(C) = \frac{9}{5}C + 32$. Prove your answer.

(5) **[20 pts]** Prove that if $m, n \in \mathbb{N}$ and there is a surjection $f : [m] \to [n]$, then $m \geq n$.

(6) **[10 pts]** Let X be a finite set with $|X| = n > 1$. Let $x \in X$ and $y \notin X$. Prove that

$$|X \setminus \{x\}| = n - 1$$

and

$$|X \cup \{y\}| = n + 1.$$

(7) **[10 pts]** Prove that if X and Y are finite sets and there exists a bijection $h : X \to Y$, then $|X| = |Y|$.