Please follow the instructions and guidelines detailed at the beginning of the first assignment.

This homework is out of 100pts.

(1) [20 pts] Let \( f : X \to Y \) and \( g : Y \to Z \). For each of the following, prove your answer.
   (a) If \( f \) is injective, can we conclude that \( g \circ f \) is injective?
   (b) If \( g \) is surjective, can we conclude that \( g \circ f \) is surjective?
   (c) If \( f \) and \( g \) are both surjective, can we conclude that \( g \circ f \) is surjective?
   (d) If \( f \) and \( g \) are both injective, can we conclude that \( g \circ f \) is injective?

(2) [20 pts] Let \( f : A \to B \) be a function. Let \( S \) and \( T \) be subsets of \( A \). If \( f \) is injective, can we conclude that \( f[S \setminus T] = f[S] \setminus f[T] \)? Prove your answer.

(3) [10 pts] Let \( p : \mathbb{N} \times \mathbb{N} \to \mathbb{N} \) be the function \( p(x, y) = 2^x(2y + 1) - 1 \). Show that \( p \) is a bijection.

(4) [10 pts] Find the inverse of the function \( f : \mathbb{R} \to \mathbb{R} \) defined by \( f(C) = \frac{2}{3}C + 32 \). Prove your answer.

(5) [20 pts] Prove that if \( m, n \in \mathbb{N} \) and there is a surjection \( f : [m] \to [n] \), then \( m \geq n \).

(6) [10 pts] Let \( X \) be a finite set with \( |X| = n > 1 \). Let \( x \in X \) and \( y \notin X \). Prove that
   \[ |X \setminus \{x\}| = n - 1 \]
   and
   \[ |X \cup \{y\}| = n + 1. \]

(7) [10 pts] Prove that if \( X \) and \( Y \) are finite sets and there exists a bijection \( h : X \to Y \), then \( |X| = |Y| \).