Please follow the instructions and guidelines detailed at the beginning of the first assignment.

You should prove everything in this assignment by induction.

This homework is out of 100pts.

(1) [**10 pts**] Prove that for all $n \geq 0$,

$$\sum_{k=0}^{n} 2^k = 2^{n+1} - 1.$$

(2) [**20 pts**] Prove that for all $n \geq 0$, $2^{2n} - 1$ is divisible by 3. (Use induction here. We'll come back later and re-prove this another way during our unit on number theory.)

(3) [**20 pts**] Show that every number greater than or equal to 15 can be represented as the sum of multiples of 3 and 7. (For example, $17 = 1 \cdot 3 + 2 \cdot 7$.)

(4) [**20pts**] Recall that the Fibonacci Numbers are defined by: $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for all $n \geq 2$.

Prove that for all $n \geq 0$, f_{4n} is a multiple of 3.

(5) [**10pts**] Recall the triangle inequality: if a and b are real numbers, then $|a + b| \leq |a| + |b|$. Show that for all $n \geq 1$,

$$\Big|\sum_{i=1}^{n} a_i\Big| \leq \sum_{i=1}^{n} |a_i|.$$

(6) [**10pts**] In class, we proved that for every $n \geq 1$, the 2^n-by-2^n checkerboard with a corner removed can be tiled by triominos. Prove that there is a tiling even if the removed square is not at the corner.

In other words, prove that if you have a 2^n-by-2^n checkerboard, for any square that you remove, you can tile the remaining checkerboard.

(7) [**10pts**] If you're not careful with your induction proofs, you can prove something completely nonsensical!

Choose one of the following and write an incorrect, but at least *slightly* convincing, proof that it is true. Then explain what is wrong with your proof.

This will be graded for completion only. Be creative! Imagine the kinds of mistakes that you could make when writing an induction proof, and how that can lead to proving something silly.

(a) Every natural number is odd.
(b) Everyone in the world is Santa Claus.
(c) For all $n \geq 0$, $n + 1 < n$.
(d) Show that for every natural number n, $7^{n+1} + 8^{2n+1}$ is divisible by 57.