Concepts of Mathematics  
Assignment #3  
Due June 1

Please follow the instructions and guidelines detailed at the beginning of the first assignment.

This homework is out of 100pts.

(1) **[20 pts]** If $A$ and $B$ are sets, we define $B \setminus A$ (“$B$ minus $A$”) to be the set $\{x \in B \mid x \notin A\}$. Prove these identities:
   
   (a) If $A_1, A_2$ and $B$ are sets then $B \setminus (A_1 \cup A_2) = (B \setminus A_1) \cap (B \setminus A_2)$.
   
   (b) If $A_1, A_2$ and $B$ are sets then $B \setminus (A_1 \cap A_2) = (B \setminus A_1) \cup (B \setminus A_2)$.
   
   (c) If $\{A_i\}_{i \in I}$ is an indexed family of sets and $B$ is a set then $B \setminus (\bigcap_{i \in I} A_i) = \bigcup_{i \in I} (B \setminus A_i)$.
   
   (d) If $\{A_i\}_{i \in I}$ is an indexed family of sets and $B$ is a set then $B \setminus (\bigcup_{i \in I} A_i) = \bigcap_{i \in I} (B \setminus A_i)$.

What do these identities remind you of?

(2) **[10 pts]** Given two sets $A$ and $B$, we define the **symmetric difference** of $A$ and $B$ (denoted $A \Delta B$) to be the set $(A \setminus B) \cup (B \setminus A)$. Prove that we can also compute $A \Delta B$ as $(A \cup B) \setminus (A \cap B)$. Next, prove that $(A \Delta B) \Delta A = B$.

(3) **[10 pts]** Let $\{A_i\}_{i \in I}$ be an indexed family of sets. Let $B$ be a set. For each of the following, explain in words how you would go about proving a statement of that form.
   
   (a) $B \subseteq \bigcup_{i \in I} A_i$
   
   (b) $B \subseteq \bigcup_{i \in I} A_i$
   
   (c) $B \notin \bigcup_{i \in I} A_i$
   
   (d) $B = \bigcup_{i \in I} A_i$

(4) **[10 pts]** Given a set $X$, the **power set of $X$**, denoted $\mathcal{P}(X)$, is the set of all subsets of $X$. In other words, $Y \in \mathcal{P}(X)$ if and only if $Y \subseteq X$.

For example, $\mathcal{P}(\{0, 1\}) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$.

Compute the following:
   
   (a) $\mathcal{P}(\{0\})$
   
   (b) $\mathcal{P}(\mathcal{P}(\{0\}))$
   
   (c) $\mathcal{P}(\mathcal{P}(\{0\})) \cap \mathcal{P}(\{0\})$
   
   (d) $\mathcal{P}(\emptyset)$

(5) **[20 pts]** Prove that $X \subseteq Y$ if and only if $\mathcal{P}(X) \subseteq \mathcal{P}(Y)$.

(6) **[20 pts]** For $n \in \mathbb{N}$, let $[n]$ denote the set $\{0, 1, ..., n\}$. Which one of the following statements is the true one? Prove your answer. (Think about this one carefully!)
   
   (a) $\mathcal{P}(\mathbb{N}) \supseteq \bigcup_{n \in \mathbb{N}} \mathcal{P}([n])$
   
   (b) $\mathcal{P}(\mathbb{N}) \subseteq \bigcup_{n \in \mathbb{N}} \mathcal{P}([n])$
   
   (c) $\mathcal{P}(\mathbb{N}) = \bigcup_{n \in \mathbb{N}} \mathcal{P}([n])$

(7) **[10 pts]** If $A$ and $B$ are sets, prove that $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

Give an example of $A$ and $B$ such that $\mathcal{P}(A) \cup \mathcal{P}(B) \nsubseteq \mathcal{P}(A \cup B)$. 

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