Please follow the instructions and guidelines detailed at the beginning of the first assignment.

This homework is out of 100pts.

(1) **[20 pts]** If A and B are sets, we define $B \setminus A$ ("B minus A") to be the set $\{x \in B \mid x \notin A\}$. Prove these identities:
   (a) If A_1, A_2 and B are sets then $B \setminus (A_1 \cup A_2) = (B \setminus A_1) \cap (B \setminus A_2)$.
   (b) If A_1, A_2 and B are sets then $B \setminus (A_1 \cap A_2) = (B \setminus A_1) \cup (B \setminus A_2)$.
   (c) If $\{A_i\}_{i \in I}$ is an indexed family of sets and B is a set then $B \setminus (\bigcap_{i \in I} A_i) = \bigcup_{i \in I}(B \setminus A_i)$.
   (d) If $\{A_i\}_{i \in I}$ is an indexed family of sets and B is a set then $B \setminus (\bigcup_{i \in I} A_i) = \bigcap_{i \in I}(B \setminus A_i)$.
   What do these identities remind you of?

(2) **[10 pts]** Given two sets A and B, we define the *symmetric difference* of A and B (denoted $A \Delta B$) to be the set $(A \setminus B) \cup (B \setminus A)$. Prove that we can also compute $A \Delta B$ as $(A \cup B) \setminus (A \cap B)$. Next, prove that $(A \Delta B)\Delta A = B$

(3) **[10 pts]** Let $\{A_i\}_{i \in I}$ be an indexed family of sets. Let B be a set. For each of the following, explain in words how you would go about proving a statement of that form.
   (a) $B \subseteq \bigcup_{i \in I} A_i$
   (b) $B \subsetneq \bigcup_{i \in I} A_i$
   (c) $B \not\subseteq \bigcup_{i \in I} A_i$
   (d) $B = \bigcup_{i \in I} A_i$

(4) **[10 pts]** Given a set X, the *power set of* X, denoted $\mathcal{P}(X)$, is the set of all subsets of X. In other words, $Y \in \mathcal{P}(X)$ iff $Y \subseteq X$.
   For example, $\mathcal{P}(\{0,1\}) = \{\emptyset, \{0\}, \{1\}, \{0,1\}\}$.
   Compute the following:
   (a) $\mathcal{P}(\{0\})$
   (b) $\mathcal{P}(\mathcal{P}(\{0\}))$
   (c) $\mathcal{P}(\mathcal{P}(\{0\})) \bigcap \mathcal{P}(\{0\})$
   (d) $\mathcal{P}(\emptyset)$

(5) **[20pts]** Prove that $X \subseteq Y$ iff $\mathcal{P}(X) \subseteq \mathcal{P}(Y)$.

(6) **[20 pts]** For $n \in \mathbb{N}$, let $[n]$ denote the set $\{0, 1, ..., n\}$. Which one of the following statements is the true one? Prove your answer. (Think about this one carefully!)
   (a) $\mathcal{P}(\mathbb{N}) \supsetneq \bigcup_{n \in \mathbb{N}} \mathcal{P}([n])$
   (b) $\mathcal{P}(\mathbb{N}) \subsetneq \bigcup_{n \in \mathbb{N}} \mathcal{P}([n])$
   (c) $\mathcal{P}(\mathbb{N}) = \bigcup_{n \in \mathbb{N}} \mathcal{P}([n])$

(7) **[10pts]** If A and B are sets, prove that

$$\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B).$$

   Give an example of A and B such that

$$\mathcal{P}(A) \cup \mathcal{P}(B) \subsetneq \mathcal{P}(A \cup B).$$