Contents

Preface 9

Acknowledgments 12

Course outlines 13

Chapter 1. Fundamentals 15

Introduction 15

1. Structures and languages 17

2. The basic concepts 29

3. On existence of models and elementary submodels 48

4. The Erdős-Rado Theorem 65

5. Applications of the compactness theorem 78

6. Skolemization 93

7. The filter of closed unbounded sets 99

8. Models of weak set theory 107

9. Ehernfeucht-Fraïssé games* 111

10. Model complete-theories* 112

11. Two applications to algebra* 120

12. Ultraproducts* 122

13. Non-standard analysis* 124

14. A mathematical introduction to the book 125

15. When does a class have a structure theory? Shelah's thesis 128

Chapter 2. Types and indicsernibles 129

Introduction 129

1. $D(T)$ the diagram of the theory T 130
2. Saturated models 139
3. More on saturated models and the monster model 148
4. More on model complete theories* 153
5. Definability 154
6. Indiscernibles and Eherenfeucht-Mostowski models 159
7. Countable models and Henkin's omitting types theorem 169
8. Types as a topological space* 179
9. More on existence, omitting types, and the completeness theorem 184
10. The Paris Harrington's theorem* 189
11. Basics of two cardinal theorems 190
12. More on omitting types 192
13. More on two cardinal theorems* 197
14. Chang's conjecture and Jónsson algebras* 200

Chapter 3. Morley's Theorem 205

Introduction 205
1. Dimension in model theory 207
2. A rank function 209
3. Existence of indiscernibles and non-splitting 214
4. $\aleph_0$-stability 217
5. Primary models 222
6. Every model is saturated 225
7. Strongly minimal sets 229
8. Some properties of $|T|^+$-categorical theories 235
9. The Baldwin Lachlan proof 240
10. Keisler's proof* 242

Chapter 4. Stability 243

Introduction 243
1. Basics 244

2. The order dichotomy 246
3. Indiscernible sequences 253
4. Noetherian topological spaces 254
5. Ranks revisited 257
6. Characterizations of stable theories 264
7. Definability of types 266
8. On the function $D[\theta(\mathbf{x};\mathbf{a}),\Delta,\mu^+]$ 269
9. Not superstable theories 286
10. The independence and strict order properties 291
11. Simple theories 296

Chapter 5. Stability in algebra 297
Introduction 297
1. Definable groups 297
2. Superstable fields are algebraically closed 298
3. Algebraic and model-theoretic dimensions 302
4. The indecomposability theorem 302
5. Model Theory of algebraically closed fields 303
6. An application to differentially closed fields* 306

Chapter 6. Forking calculus in simple theories 307
Introduction 307
1. General notion of independence 308
2. Dividing and forking 316
3. Ranks and Simple Theories 325
4. Forking in Simple Theories 332
5. Shelah's Boolean Algebra 341
6. Semi simple theories 349

Chapter 7. More stability 351
Introduction 351

1. Finite equivalence relations theorem 351
2. The stability spectrum theorem 351
3. Chains of saturated models 351
4. Canonical bases and $\mathfrak{C}^{eq}$ 351

Chapter 8. Prime models 353
Introduction 353
1. Isolation notions and existence 353
2. Uniqueness 356

Chapter 9. Orthogonality calculus 359
Introduction 359
1. Regular types 359

Chapter 10. Morley's theorem for uncountable theories 361
Introduction 361
1. Weakly minimal formulas 361

Chapter 11. The Hrushovski-Zilber group configuration 363
Introduction 363
1. Basics 363

Chapter 12. Classification theory of non elementary classes 365
Introduction 365
1. Fundamentals 368
2. $\mathcal{K}$-embeddings 376
3. A substitute for saturation 383
4. Abstract classes are PC-classes 385
5. The amalgamation property and the weak diamond 388
6. On pseudo elementary classes 413
7. Types 427

Chapter 13. Non structure theory 437

Introduction 437

1. Unstable theories 437

2. Unsuperstable theories and Boolean algebras 437

Chapter 14. Survey 439

Introduction 439

1. The main gap (Shelah's great theorem) 439

2. Classification theory for non-elementary classes 441

3. Geometric stability (or the fine structure theory) 441

4. Lang-Mordel 441

5. Ax and Kochen 441

6. o-minimal theories 441

7. Soft model theory 441

8. Finite model theory 441

9. Non standard analysis 441

Chapter 15. Appendix on combinatorial geometry 443

Introduction 443

1. Pregeometries (or Matroids) 443

2. Abstract dependence 449

3. Projective geometries 449

Chapter 16. A miniguide to the literature 451

Bibliography 453

Index 459