

Putnam E.2

Po-Shen Loh

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1 Problems

Putnam 2005/A1. Show that every positive integer is a sum of one or more numbers of the form $2^r 3^s$, where r and s are nonnegative integers and no summand divides another. (For example, $23 = 9 + 8 + 6$.)

Putnam 2005/A2. Let $\mathbf{S} = \{(a, b) : a = 1, 2, \dots, n, b = 1, 2, 3\}$. A *rook tour* of $\mathbf{S}$ is a polygonal path made up of line segments connecting points $p_1, p_2, \dots, p_{3n}$ in sequence such that

- (i) $p_i \in \mathbf{S}$,
- (ii) p_i and p_{i+1} are a unit distance apart, for $1 \leq i < 3n$,
- (iii) for each $p \in \mathbf{S}$ there is a unique i such that $p_i = p$.

How many rook tours are there that begin at $(1, 1)$ and end at $(n, 1)$?

Putnam 2005/A3. Let $p(z)$ be a nonconstant polynomial of degree n , all of whose zeros have absolute value 1 in the complex plane. Put $g(z) = p(z)/z^{n/2}$. Show that all zeros of $g'(z) = 0$ have absolute value 1.