# Putnam Σ.13

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## 1 Problems

**Putnam 1999/B4.** Let $f$ be a real function with a continuous third derivative such that $f(x), f'(x), f''(x), f'''(x)$ are positive for all $x$. Suppose that $f'''(x) \leq f(x)$ for all $x$. Show that $f'(x) < 2f(x)$ for all $x$.

**Putnam 1999/B5.** For an integer $n \geq 3$, let $\theta = 2\pi/n$. Evaluate the determinant of the $n \times n$ matrix $I + A$, where $I$ is the $n \times n$ identity matrix and $A = (a_{jk})$ has entries $a_{jk} = \cos(j\theta + k\theta)$ for all $j, k$.

**Putnam 1999/B6.** Let $S$ be a finite set of integers, each greater than 1. Suppose that for each integer $n$ there is some $s \in S$ such that $\gcd(s, n) = 1$ or $\gcd(s, n) = s$. Show that there exist $s, t \in S$ such that $\gcd(s, t)$ is prime.