

# Putnam $\Sigma.12$

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## 1 Problems

**Putnam 1999/A4.** Sum the series

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2 n}{3^m (n3^m + m3^n)}.$$

**Putnam 1999/A5.** Prove that there is a constant  $C$  such that, if  $p(x)$  is a polynomial of degree 1999, then

$$|p(0)| \leq C \int_{-1}^1 |p(x)| dx.$$

**Putnam 1999/A6.** The sequence  $(a_n)_{n \geq 1}$  is defined by  $a_1 = 1, a_2 = 2, a_3 = 24$ , and, for  $n \geq 4$ ,

$$a_n = \frac{6a_{n-1}^2 a_{n-3} - 8a_{n-1} a_{n-2}^2}{a_{n-2} a_{n-3}}.$$

Show that, for all  $n$ ,  $a_n$  is an integer multiple of  $n$ .