

# Putnam E.11

Po-Shen Loh

14 November 2023

## 1 Problems

**Putnam 2014/A1.** Prove that every nonzero coefficient of the Taylor series of

$$(1 - x + x^2)e^x$$

about  $x = 0$  is a rational number whose numerator (in lowest terms) is either 1 or a prime number.

**Putnam 2014/A2.** Let  $A$  be the  $n \times n$  matrix whose entry in the  $i$ -th row and  $j$ -th column is

$$\frac{1}{\min(i, j)}$$

for  $1 \leq i, j \leq n$ . Compute  $\det(A)$ .

**Putnam 2014/A3.** Let  $a_0 = 5/2$  and  $a_k = a_{k-1}^2 - 2$  for  $k \geq 1$ . Compute

$$\prod_{k=0}^{\infty} \left(1 - \frac{1}{a_k}\right)$$

in closed form.