Putnam Σ.9

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1 Problems

Putnam 2012/B4. Suppose that $a_0 = 1$ and that $a_{n+1} = a_n + e^{-a_n}$ for $n = 0, 1, 2, \ldots$. Does $a_n - \log n$ have a finite limit as $n \to \infty$? (Here $\log n = \log_e n = \ln n$.)

Putnam 2012/B5. Prove that, for any two bounded functions $g_1, g_2 : \mathbb{R} \to [1, \infty)$, there exist functions $h_1, h_2 : \mathbb{R} \to \mathbb{R}$ such that, for every $x \in \mathbb{R}$,

$$\sup_{s \in \mathbb{R}} (g_1(s)^x g_2(s)) = \max_{t \in \mathbb{R}} (x h_1(t) + h_2(t)).$$

Putnam 2012/B6. Let p be an odd prime number such that $p \equiv 2 \pmod 3$. Define a permutation π of the residue classes modulo p by $\pi(x) \equiv x^3 \pmod p$. Show that π is an even permutation if and only if $p \equiv 3 \pmod 4$.