Putnam $\Sigma.7$

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1 Problems

Putnam 1997/B4. Let $a_{m,n}$ denote the coefficient of x^n in the expansion of $(1+x+x^2)^m$. Prove that for all integers $k \geq 0$,

$$0 \leq \sum_{i=0}^{\lfloor \frac{2k}{3} \rfloor} (-1)^i a_{k-i,i} \leq 1.$$

Putnam 1997/B5. Prove that for $n \geq 2$,

$$\overbrace{2^{2^{\cdots^{2}}}}^{n \text{ terms}} \equiv \overbrace{2^{2^{\cdots^{2}}}}^{n-1 \text{ terms}} \pmod{n}.$$

Putnam 1997/B6. The dissection of the 3–4–5 triangle into four congruent right triangles similar to the original has diameter 5/2. (The diameter of a dissection is the least upper bound of the distances between pairs of points belonging to the same part.) Find the least diameter of a dissection of this triangle into four parts.