

# Putnam $\Sigma.4$

Po-Shen Loh

20 September 2020

## 1 Problems

**Putnam 1998/B4.** Find necessary and sufficient conditions on positive integers  $m$  and  $n$  so that

$$\sum_{i=0}^{mn-1} (-1)^{\lfloor i/m \rfloor + \lfloor i/n \rfloor} = 0.$$

**Putnam 1998/B5.** Let  $N$  be the positive integer with 1998 decimal digits, all of them 1; that is,

$$N = 1111 \cdots 11.$$

Find the thousandth digit after the decimal point of  $\sqrt{N}$ .

**Putnam 1998/B6.** Prove that, for any integers  $a, b, c$ , there always exists a positive integer  $n$  such that  $\sqrt{n^3 + an^2 + bn + c}$  is not an integer.