

Putnam E.15

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1 Problems

Putnam 1981/B1. Evaluate

$$\lim_{n \rightarrow \infty} \frac{1}{n^5} \sum_{r=1}^n \sum_{s=1}^n 5r^4 - 18r^2s^2 + 5s^4.$$

Putnam 1981/B2. What is the minimum value of

$$(a-1)^2 + \left(\frac{b}{a} - 1\right)^2 + \left(\frac{c}{b} - 1\right)^2 + \left(\frac{4}{c} - 1\right)^2$$

over all real numbers a, b, c satisfying $1 \leq a \leq b \leq c \leq 4$?

Putnam 1981/B3. Prove that infinitely many positive integers n have the property that for any prime p dividing $n^2 + 3$, we can find an integer m such that both p divides $m^2 + 3$ and $m^2 < n$.