

# Putnam E.13

Po-Shen Loh

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## 1 Problems

**Putnam 1982/B1.**  $ABC$  is an arbitrary triangle, and  $M$  is the midpoint of  $BC$ . How many pieces are needed to dissect  $AMB$  into triangles which can be reassembled to give  $AMC$ ?

**Putnam 1982/B2.** Let  $a(r)$  be the number of lattice points inside the circle centered at the origin, with radius  $r$ . Let

$$k = 1 + e^{-1} + e^{-4} + \dots + e^{-n^2} + \dots$$

Express

$$\int_{\mathbb{R}^2} a(\sqrt{x^2 + y^2}) e^{-(x^2 + y^2)} dx dy$$

as a polynomial in  $k$ .

**Putnam 1982/B3.** Let  $p_n$  be the probability that two numbers selected independently and randomly from  $\{1, 2, 3, \dots, n\}$  have a sum which is a square. Find  $\lim_{n \rightarrow \infty} p_n \sqrt{n}$ .