Putnam E.11

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1 Problems

Putnam 1983/B2. Let $f(n)$ be the number of ways of representing n as a sum of powers of 2 with no power being used more than 3 times. For example, $f(7) = 4$ (the representations are $4 + 2 + 1$, $4 + 1 + 1 + 1$, $2 + 2 + 2 + 1$, $2 + 2 + 1 + 1 + 1$). Can we find a real polynomial $p(x)$ such that $f(n) = \lfloor p(n) \rfloor$?

Putnam 1983/A3. Let $f(n) = 1 + 2n + 3n^2 + \cdots + (p - 1)n^{p-2}$, where p is an odd prime. Prove that if $f(m) = f(n) \pmod{p}$, then $m = n \pmod{p}$.

Putnam 1983/B3. Let y_1, y_2, and y_3 be solutions of $y''' + a(x)y'' + b(x)y' + c(x)y = 0$ such that $y_1^2 + y_2^2 + y_3^2 = 1$ for all x. Find constants α and β such that $y_1'(x)^2 + y_2'(x)^2 + y_3'(x)^2$ is a solution of $y' + \alpha a(x)y + \beta c(x) = 0$.