# Putnam E.7

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## 1 Problems

**Putnam 1985/A1.** Determine, with proof, the number of ordered triples $(A_1, A_2, A_3)$ of sets which have the property that

**(i)** $A_1 \cup A_2 \cup A_3 = \{1, 2, \ldots, 10\}$, and

**(ii)** $A_1 \cap A_2 \cap A_3 = \emptyset$.

Express the answer in the form $2^a 3^b 5^c 7^d$, where $a$, $b$, $c$, and $d$ are nonnegative integers.

**Putnam 1985/A2.** Let $T$ be an acute triangle. Inscribe a rectangle $R$ in $T$ such that the bottom edge of $R$ is on the base of $T$, and the two top corners of $R$ touch the sides of $T$. Inscribe another rectangle $S$ by placing the bottom edge of $S$ on the top edge of $R$, and the top corners of $S$ on the sides of $T$. Let $A(X)$ denote the area of polygon $X$. Find the maximum value, or show that no maximum exists, of $\frac{A(R)+A(S)}{A(T)}$, where $T$ ranges over all triangles and $R, S$ over all rectangles.

**Putnam 1985/A3.** Let $d$ be a real number. For each integer $m \geq 0$, define a sequence $\{a_m(j)\}$, $j = 0, 1, 2, \ldots$ by the condition

$$a_m(0) = d/2^m, \quad \text{and} \quad a_m(j+1) = (a_m(j))^2 + 2a_m(j), \quad j \geq 0.$$

Evaluate $\lim_{n\to\infty} a_n(n)$.