Putnam E.6

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1 Problems

Putnam 1986/B1. Inscribe a rectangle of base b and height h in a circle of radius one. Further inscribe an isosceles triangle of base b between the b-side of the rectangle and the minor arc of the circle that it determines. For what value of h do the rectangle and triangle have the same area?

Putnam 1986/B2. Prove that there are only a finite number of possibilities for the ordered triple $T = (x-y, y-z, z-x)$, where x, y, and z are complex numbers satisfying the simultaneous equations

$$x(x-1) + 2yz = y(y-1) + 2zx = z(z-1) + 2xy\,,$$

and list all such triples T.

Putnam 1986/B3. Let Γ consist of all polynomials in x with integer coefficients. For f and g in Γ and m a positive integer, let $f \equiv g \pmod{m}$ mean that every coefficient of $f - g$ is an integral multiple of m. Let n and p be positive integers with p prime. Given that f, g, h, r, and s are in Γ with $rf + sg \equiv 1 \pmod{p}$ and $fg \equiv h \pmod{p}$, prove that there exist F and G in Γ with $F \equiv f \pmod{p}$, $G \equiv g \pmod{p}$, and $FG \equiv h \pmod{p^n}$.