1 Problems

**Putnam 2000/B4.** Let $f(x)$ be a continuous function such that $f(2x^2 - 1) = 2xf(x)$ for all $x$. Show that $f(x) = 0$ for $-1 \leq x \leq 1$.

**Putnam 2000/B5.** Let $S_0$ be a finite set of positive integers. We define finite sets $S_1, S_2, \ldots$ of positive integers as follows: the integer $a$ is in $S_{n+1}$ if and only if exactly one of $a - 1$ or $a$ is in $S_n$. Show that there exist infinitely many integers $N$ for which $S_N = S_0 \cup \{N + a : a \in S_0\}$.

**Putnam 2000/B6.** Let $B$ be a set of more than $2^{n+1}/n$ distinct points with coordinates of the form $(\pm 1, \pm 1, \ldots, \pm 1)$ in $n$-dimensional space with $n \geq 3$. Show that there are three distinct points in $B$ which are the vertices of an equilateral triangle.