1 Problems

**Putnam 2014/B1.** A base 10 over-expansion of a positive integer $N$ is an expression of the form

$$N = d_k10^k + d_{k-1}10^{k-1} + \cdots + d_010^0$$

with $d_k \neq 0$ and $d_i \in \{0, 1, 2, \ldots, 10\}$ for all $i$. For instance, the integer $N = 10$ has two base 10 over-expansions: $10 = 10 \cdot 10^0$ and the usual base 10 expansion $10 = 1 \cdot 10^1 + 0 \cdot 10^0$. Which positive integers have a unique base 10 over-expansion?

**Putnam 2014/B2.** Suppose that $f$ is a function on the interval $[1, 3]$ such that $-1 \leq f(x) \leq 1$ for all $x$ and $\int_1^3 f(x) \, dx = 0$. How large can $\int_1^3 \frac{f(x)}{x} \, dx$ be?

**Putnam 2014/B3.** Let $A$ be an $m \times n$ matrix with rational entries. Suppose that there are at least $m + n$ distinct prime numbers among the absolute values of the entries of $A$. Show that the rank of $A$ is at least 2.