1 Problems

**Putnam 1990/B1.** Find all real-valued continuously differentiable functions \( f \) on the real line such that for all \( x \),

\[
(f(x))^2 = \int_0^x [(f(t))^2 + (f'(t))^2] \, dt + 1990.
\]

**Putnam 1990/B2.** Prove that for \(|x| < 1, |z| > 1,\)

\[
1 + \sum_{j=1}^{\infty} (1 + x^j)P_j = 0,
\]

where \( P_j \) is

\[
\frac{(1 - z)(1 - zx)(1 - zx^2) \cdots (1 - zx^{j-1})}{(z - x)(z - x^2)(z - x^3) \cdots (z - x^j)}.
\]

**Putnam 1990/B3.** Let \( S \) be a set of \( 2 \times 2 \) integer matrices whose entries \( a_{ij} \) (1) are all squares of integers and, (2) satisfy \( a_{ij} \leq 200 \). Show that if \( S \) has more than \( 50387 = 15^4 - 15^2 - 15 + 2 \) elements, then it has two elements that commute.