

15. 

(Just do it)

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CMU Putnam Seminar, Fall 2019

1 Problems

1. Please go to Scaife Hall room 125 on Sat Dec 7 by 9:45am. There will be pizza (eventually).
2. You are walking from the NE corner of 14th St and 8th Ave to the SW corner of 16th St and 6th Ave. Which direction should you walk first? (Does it even make a difference?)
3. Two people are together, with one bicycle, and they are traveling to the same destination. How should they travel, so that as soon as possible, both of them have arrived at the destination?
4. We alternate writing down digits after a decimal point, thereby producing a real number between 0 and 1. You win if the number is irrational. Can you force a win?
5. To split a drink two ways, you let the first person divide it into what they think are two equal parts, and then let the second person choose one of them. Both are then satisfied. How can you do this with three people?
6. There is a deck of 52 cards, of which 13 happen to be face-up and the other 39 are face-down. You don't know which of them are face-up, and you are blindfolded. It is possible for you to make 2 piles of cards such that every card appears in exactly one of the piles, every card is either face-up or face-down, and the number of face-up cards in both piles is exactly the same.
7. Devise the smallest plane set such that no point is at a rational distance from all points of the set.
8. Given an infinite number of points in the plane with all the mutual distances integers, prove that the points are all collinear.
9. Given that $f(x)$ increases from 0 to 1 as x does, prove that the graph of $y = f(x)$ between $0 \leq x \leq 1$ can be covered by n rectangles with sides parallel to the axes and each having area $\frac{1}{n^2}$.
10. Given a simple plane arc of length more than 1, prove that for some n there are more than n points on the arc whose mutual distances are all at least $1/n$.

2 Homework

None of these problems need to be written up, because this is the last week of the semester. If you have any homework that has not yet been turned in, please turn it in before the last day of class, into my mailbox in the math department (Wean 6113). See you at the Putnam Competition!