

# Putnam E.10

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## 1 Problems

**Putnam 2001/A1.** Consider a set  $S$  and a binary operation  $*$ , i.e., for each  $a, b \in S$ ,  $a * b \in S$ . Assume  $(a * b) * a = b$  for all  $a, b \in S$ . Prove that  $a * (b * a) = b$  for all  $a, b \in S$ .

**Putnam 2001/A2.** You have coins  $C_1, C_2, \dots, C_n$ . For each  $k$ ,  $C_k$  is biased so that, when tossed, it has probability  $1/(2k + 1)$  of falling heads. If the  $n$  coins are tossed, what is the probability that the number of heads is odd? Express the answer as a rational function of  $n$ .

**Putnam 2001/A3.** For each integer  $m$ , consider the polynomial

$$P_m(x) = x^4 - (2m + 4)x^2 + (m - 2)^2.$$

For what values of  $m$  is  $P_m(x)$  the product of two non-constant polynomials with integer coefficients?