Putnam Σ.13

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1 Problems

Putnam 1993/B4. The function $K(x, y)$ is positive and continuous for $0 \leq x \leq 1, 0 \leq y \leq 1$, and the functions $f(x)$ and $g(x)$ are positive and continuous for $0 \leq x \leq 1$. Suppose that for all x, $0 \leq x \leq 1$,

$$\int_0^1 f(y)K(x, y)\, dy = g(x)$$

and

$$\int_0^1 g(y)K(x, y)\, dy = f(x).$$

Show that $f(x) = g(x)$ for $0 \leq x \leq 1$.

Putnam 1993/B5. Show there do not exist four points in the Euclidean plane such that the pairwise distances between the points are all odd integers.

Putnam 1993/B6. Let S be a set of three, not necessarily distinct, positive integers. Show that one can transform S into a set containing 0 by a finite number of applications of the following rule: Select two of the three integers, say x and y, where $x \leq y$ and replace them with $2x$ and $y - x$.