Putnam $\Sigma.2$

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4 September 2016

1 Problems

Putnam 1998/B4. Find necessary and sufficient conditions on positive integers m and n so that

$$\sum_{i=0}^{mn-1} (-1)^{\lfloor i/m \rfloor + \lfloor i/n \rfloor} = 0.$$

Putnam 1998/B5. Let N be the positive integer with 1998 decimal digits, all of them 1; that is,

$$N = 1111 \cdots 11.$$

Find the thousandth digit after the decimal point of $\sqrt{N}$.

Putnam 1998/B6. Prove that, for any integers a, b, c, there always exists a positive integer n such that $\sqrt{n^3 + an^2 + bn + c}$ is not an integer.