

# Putnam E.10

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## 1 Problems

**Putnam 1984/B1.** Define  $f(n) = 1! + 2! + \cdots + n!$ . Find a recurrence relation  $f(n+2) = a(n)f(n+1) + b(n)f(n)$ , where  $a(x)$  and  $b(x)$  are polynomials.

**Putnam 1984/B2.** Find the minimum of  $f(x, y) = (x - y)^2 + (\sqrt{2 - x^2} - \frac{y}{y})^2$  in the half-infinite strip  $0 < x < \sqrt{2}, y > 0$ .

**Putnam 1984/B3.** Let  $S$  be a set with  $n$  elements. Can we find a binary operation  $\star$  on  $S$  which satisfies (1) right cancellation:  $a \star c = b \star c$  implies  $a = b$  (for all  $a, b, c$ ), and (2) total non-associativity:  $a \star (b \star c) \neq (a \star b) \star c$  for all  $a, b, c$ ? Note that we are not just requiring that  $\star$  is not associative, but that it is never associative.