Putnam E.4

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1 Problems

Putnam 1987/B1. Evaluate

$$\int_2^4 \frac{\sqrt{\ln(9-x)}dx}{\sqrt{\ln(9-x)} + \sqrt{\ln(x+3)}}.$$

Putnam 1987/B2. Let r, s, and t be integers with $0 \le r$, $0 \le s$, and $r + s \le t$. Prove that

$$\frac{\binom{s}{0}}{\binom{t}{r}} + \frac{\binom{s}{1}}{\binom{t}{r+1}} + \cdots + \frac{\binom{s}{s}}{\binom{t}{r+s}} = \frac{t+1}{(t+1-s)\binom{t-s}{r}}.$$

Putnam 1987/B3. Let F be a field in which $1 + 1 \ne 0$. Show that the set of solutions to the equation $x^2 + y^2 = 1$ with x and y in F is given by $(x, y) = (1, 0)$ and

$$(x, y) = \left(\frac{r^2 - 1}{r^2 + 1}, \frac{2r}{r^2 + 1} \right),$$

where r runs through the elements of F such that $r^2 \ne -1$.