

# Putnam E.02

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## 1 Problems

**Putnam 1988/B1.** A *composite* (positive integer) is a product  $ab$  with  $a$  and  $b$  not necessarily distinct integers in  $\{2, 3, 4, \dots\}$ . Show that every composite is expressible as  $xy + xz + yz + 1$ , with  $x, y, z$  positive integers.

**Putnam 1988/B2.** Prove or disprove: If  $x$  and  $y$  are real numbers with  $y \geq 0$  and  $y(y + 1) \leq (x + 1)^2$ , then  $y(y - 1) \leq x^2$ .

**Putnam 1988/B3.** For every  $n$  in the set  $\mathbb{N} = \{1, 2, \dots\}$  of positive integers, let  $r_n$  be the minimum value of  $|c - d\sqrt{3}|$  for all nonnegative integers  $c$  and  $d$  with  $c + d = n$ . Find, with proof, the smallest positive real number  $g$  with  $r_n \leq g$  for all  $n \in \mathbb{N}$ .